Application of the Optimisation Methods in Designing Logistic Systems

Kumbim Shala and Altin Dorri

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Kumbim SHALA*, Altin DORRI+

*University of Pristina, Faculty of Mechanical Engineering,
St.Agim Ramadani, nn, 10000, Pristina, Republic of Kosovo
kumbim.shala@uni-pr.edu

+Polytechnic University of Tirana, Faculty of Mechanical Engineering
Bulevardi Dëshmorët e Kombit Nr. 4, Tirana, Albania
adorri@fim.edu.al

Abstract— **A lot of studies in different fields are researching, creating and developing principles, methods and tools to find the solutions which are possible for all tasks and problems that occur in management, technical, production and other systems.**

In practice, we might be facing with different situations where several ways to manage them are in disposal. The aim is to find the optimal solution, which depending on the situation can be cost effective, will realise the highest possible profit, and will use the shortest possible time to reach the main goals through optimal usage of all human, technical and financial capacities. These solutions will be the main preconditions prior to undertake the managerial decision, its implementation, continuity or in planning activities.

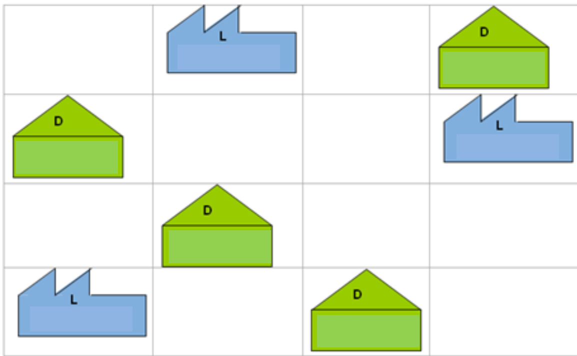
In this paper we will try to find the optimal way to design the production systems for small enterprises, in order to find the best appropriate solution that fulfils the economic and technological criteria for material, energy and information flow, including production capacities based on the market needs and the flexibility of labour force.

Our aim is to apply variety of methods for finding the initial and optimal solutions for the problem of transport, by comparing different methods, including well known degeneration methods to eliminate the obstacles that arise during the implementation of different activities for finding the optimal solution.

Keywords— **design, logistics system, the problem of transport, optimal solution, the degeneration problem, optimisation, etc.**

I. INTRODUCTION

Transportation problem has to do with finding transportation plan with lower costs for the transfer of stock (goods) from production centers (factories) to the centers of destination (Consumer. e.g. any company may have three factories which produce the same products and four destination centre's that require these products, as described in Fig.1. Transport model can be used to determine how to distribute products from factories to destination centers, so that transportation costs are minimal. Usually such analysis of the problem as a result has a transportation plan for a certain time period (days, weeks). Now, when the plan is realized as such will not change as long as any of the parameters of the problem varies (supply, demand, unit price).



Consumer / Factories	D_1	D_2	D_3	D_4	supply a_i
L_1	4	7	7	1	100
L_2	12	3	8	8	200
L_3	8	10	16	5	150
b_j	80	90	120	160	450

Fig. 1 The logistic distribution of products from its origin (factories) to destination (costumer).

From the table we see that the field (1.4) has minimal costs, so:

$$c_{14} = \min_{ij}(c_{ij}) = 1$$

$$x_{14} = \min_{14}(a_1, b_4) = \min_{14}(100{,}160) = 100 \qquad .$$

Therefore, we accomplish customer demand and in the first row, we have:

$$x_{11} = x_{12} = x_{13} = 0$$

In order fully to the same act with other customers, while continuing with the field (3.4) that costs less.

$$c_{34} = 5$$

$$x_{34} = \min_{34}(a_3, b_4 - x_{11}) = \min_{34}(150{,}160 - 100) = 60$$

and so on, we win the initial solution with minimum cost method.

Customer / Factories	D_1	D_2	D_3	D_4	supply a_i
L_1	4	7	7	1 $\boxed{100}$	100
L_2	12	3 $\boxed{90}$	8 $\boxed{110}$	8	200
L_3	8 $\boxed{80}$	10	16 $\boxed{10}$	5 $\boxed{60}$	150
b_j	80	90	120	160	450

Not-degenerated solution is basic because it met the condition

$$m+n-1=3+4-1=6.$$

The function of the goal has the value.

$$x_{11}=0 \qquad x_{14}=100$$

$$x_{12}=x_{13}=x_{21}=x_{24}=x_{32}=0 \qquad x_{22}=90$$

$$x_{23}=110 \qquad x_{33}=10$$

$$x_{31}=80 \qquad x_{34}=60$$

$$F(x)=c_{12}x_{12}+c_{13}x_{13}+c_{14}x_{14}+c_{21}x_{21}+c_{22}x_{22}+c_{34}x_{34}$$

$$F(x)=1\cdot100+3\cdot90+8\cdot110+8\cdot80+16\cdot10+5\cdot60=2350\ Euro$$

In addition we have to verify that the solution with minimum cost method is the optimal solution. Will do with the method of jump from stone to stone, during interpretation of this method we based in five acts.

First action:

$d_{ij}=c_{ij}-z_{ij}$, $x_{ij}=0$,

For $x_{11}=0$ (this field programmed without transport cost $c_{11}=4$)

$$d_{11}=c_{11}-z_{11}=c_{11}-c_{14}+c_{34}-c_{41}=4-1+5-8=0$$

and so on

$d_{12}=0$; $d_{13}=-5$; $d_{21}=12$; $d_{24}=11$ $d_{32}=-1$

Second action:

After determining the indicators d_{ij}, according to the criteria, stems that condition is not fulfilled, since the two indicators are $d_{ij}\le 0$. For this reason, the next move will continue to improve the program.

Customer / Factories	D_1	D_2	D_3	D_4	Supply a_i
L_1	4 **0**	7 **0**	7 **-5**	1 $\boxed{100}$	100
L_2	12 **12**	3 $\boxed{90}$	8 $\boxed{110}$	8 **11**	200
L_3	8 $\boxed{80}$	10 **-1**	16 $\boxed{10}$	5 $\boxed{60}$	150
b_j	80	90	120	160	450

Third Action:

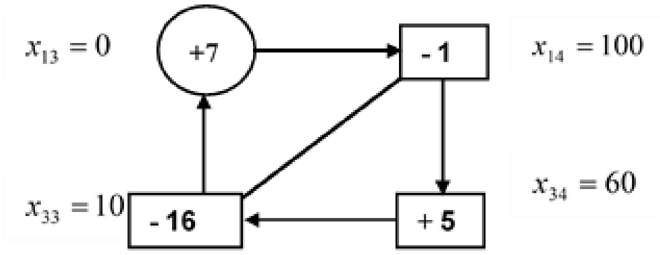
Improving program for negative component or for more efficient direction, defined with this criteria:

$\max\limits_{ij}|d_{ij}|$, for $d_{ij}<0$ $(i=1,2,...,m)$ and $(j=1,2,...,n)$

$$|d_{13}| \quad |d_{32}|$$

$$|-5| \quad |-1|$$

More efficient direction or component which should be improved is $d_{13}=-5$. After that follows the new program for the programmed field.



Fourth action:

The program of transport quantity

$\Theta=\min\limits_{ij}(x_{ij})>0$ $(i=1,2,...,m)$ and $(j=1,2,...,n)$

$\Theta=\min\limits_{ij}(x_{14},x_{33})=(100,10)=10$ - for negative field

Action fifth:

Calculation of the new program based on the rule (criterion)

$x'_{ij}=x_{ij}\pm\Theta$ $(i=1,2,...,m)$ and $(j=1,2,...,n)$

The new program:

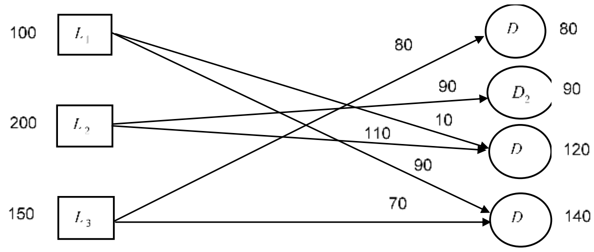
$$x'_{13}=x_{13}+\Theta=0+10=10$$

$$x'_{14}=x_{14}-\Theta=100-10=90$$

$$x'_{34}=x_{34}+\Theta=60+10=70$$

$$x'_{33}=x_{33}-\Theta=10-10=0$$

customer / factories	D_1	D_2	D_3	D_4	supply a_i
L_1	4 **0**	7 **5**	7 $\boxed{10}$	1 $\boxed{90}$	100
L_2	12 **7**	3 $\boxed{90}$	8 $\boxed{110}$	8 **6**	200
L_3	8 $\boxed{80}$	10 **4**	16 **5**	5 $\boxed{70}$	150
b_j	80	90	120	160	450



Rules for determining the possible solutions is:

$m+n-1=3+4-1=6$, so this is non-degenerated solution $F_2(x)=2300 Euro$

Continue with the determination of free fields, ie. back in action first:

$$d_{11}=0 \quad d_{12}=5 \quad d_{21}=7 \quad d_{32}=4 \quad d_{33}=5$$

Since all indicators calculated values are positive can then be concluded that the solution is optimal, but for the indicator $d_{11}=0$ can be found the other alternative optimal solution by increasing the effects on existing conditions, but without affecting the value of the goal function.

The Modified Method (MODI)

The method of modifying or Modi method serves to determine the optimal solution, but after the initial solution is determined by any methods listed.

For this method, the necessary number of iteration is smaller towards achieving optimal solution. This method is also called alternative method of potential due to the involvement potentials u_i and v_j .

Solving the transportation problem with method of potentials will becomes after the initial plan is made and calculated costs of transportation under that plan is done.

First action:

$$c_{ij}=u_i+v_j \quad x_{ij}>0 \qquad (i=1,2,3)\,,\ \ (j=1,2,...n)$$

Customer Factory	D_1	D_2	D_3	D_4	Oferta a_i	u_i
L_1	4	7	7	1 $\boxed{100}$	100	u_1=0
L_2	12	3 $\boxed{90}$	8 $\boxed{110}$	8	200	u_2=-4
L_3	8 $\boxed{80}$	10	16 $\boxed{10}$	5 $\boxed{60}$	150	u_3=4
b_j	80	90	120	160	450	
v_j	v_1=4	v_2=7	v_3=12	v_4=1		

In **Second action**, determined the table for free fields along the lines:

$$x_{ij}=0 \qquad\qquad d_{ij}=c_{ij}-(u_i+v_j)$$

$$x_{11}=0 \qquad d_{11}=c_{11}-(u_1+v_1)=4-(0+4)=0$$
$$x_{12}=0 \qquad d_{12}=c_{12}-(u_1+v_2)=7-(0+7)=0$$
$$x_{13}=0 \qquad d_{13}=c_{13}-(u_1+v_3)=7-(0+12)=-5$$
$$x_{21}=0 \qquad d_{21}=c_{21}-(u_2+v_1)=12-(-4+4)=12$$
$$x_{24}=0 \qquad d_{24}=c_{24}-(u_2+v_4)=8-(-4+1)=11$$
$$x_{32}=0 \qquad d_{32}=c_{32}-(u_3+v_2)=10-(4+7)=-1$$

As is seen the values of the indicators are fully identical to the method of jumping from stone to stone, therefore and the procedure of solving the task and results with MODI method are fully the same.

Special case

Not all transportation problems are neat. Occurrence and occasions can be presented when we have an obstacle in finding activities for the initial and optimal solutions. This means that some action must be conducted before the preliminary settlement reached. Two irregularities that appear most frequently are cases when offer is not equal to demand (the problem of open transport) and the appearance of degeneration. Degeneration as a barrier, not to do with the composition of the economic category, respectively product and other elements, but with lack of the activity that is necessary to transform the system from the starting point of the initial solution to the final solution. Therefore, first we eliminate such phenomenon, and then framing methods to improve the program and calculate the optimal program.

Degeneration as a barrier may be present in two ways:

I. Degeneracy can observe during the implementation of rules for determining the basic solutions to the problem of transport $m+n-1$ for $x_{ij}>0$ solutions for non degenerated where $(i=1,2,...,m),\ \ (j=1,2,...,n)$ and $m+n-2,-3,-...,$ for $x_{ij}>0$ degenerated solutions where $(i=1,2,...,m),\ (j=1,2,...,n)$.

II. Degeneracy can be noticed if the problem tractable. At least two partial and amount (which was simbolizuan respectively were reviewed for the meaning of their offers and requests) are numerically equal, the same.
$a_i=b_j$ where $(i=1,2,...,m),(j=1,2,...,n)$

These two problems will illustrate as follows by an example.
An economic entity in the area of transport is entrusted with the transport of products stored in three distributive centers marked with L_i , in two destination centers marked with D_j. Transportation quantities X_{ij}, which must be transported from the center *i* to center *j*, are given by the proportion:

3Li ↔ A1 = 35 A2 = 50 A3 = 30
2Dj ↔ B1 = 55 B2 = 25

- Initial distribution determined by the method of minimal cost, and
- Solutions with the method of jumping from stone to stone.

This is the opened model $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \Rightarrow 115 > 80$ and thus become the model closure adding a fictitious destination center with transportation cost per unit equal to zero.

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 115 - 80 = 35$$

$$(\min); F(x) = 2x_{11} + 2x_{12} + 0x_{13} + 4x_{21} + x_{22} + 0x_{23} + 3x_{32} + 0x_{33}$$

$$x_{11} + x_{12} + x_{13} = 35 \qquad x_{11} + x_{21} + x_{31} = 50$$
$$x_{21} + x_{22} + x_{23} = 50 \qquad x_{12} + x_{22} + x_{32} = 55$$
$$x_{31} + x_{32} + x_{33} = 30 \qquad x_{13} + x_{23} + x_{33} = 35$$

Nonnegative hypothesis $x_{ij} \geq 0$ for $(i = 1,2,3)$ and $(j = 1,2,3)$

	D_1	D_2	D_3	a_i
L_1	2	2	0 $\boxed{35}$	35
L_2	4 $\boxed{25}$	1 $\boxed{25}$	0	50
L_3	1 $\boxed{30}$	3	0	30
b_j	55	25	35	115

– Initial distribution with minimum cost method
– The choice is degenerated because we have only four programmed full field
– Degeneracy can be noticed even with rule $a_1 = b_3$
– With rule $m + n - 1$, has degeneracy it with Rule $a_i = b_j$, degeneracy presented and eliminated

Now we eliminate the degeneracy with method of coefficients.
For $m = n$ can do to eliminate for row or column.

$$a_i' = a_i + a_i \frac{1}{2m} \cdot 0.5$$

the model is open and therefore, should close it

$$\sum_{i=1}^{m} a_i = a_1' + a_2' + a_3' = 37 + 54 + 32 = 123$$

$$\sum_{j=1}^{n} b_j = 115 \qquad \sum_{i=1}^{m} a_i' > \sum_{j=1}^{n} b_j$$

Because 123>115

$$b_{n+1} = \sum_{i=1}^{m} a_i' - \sum_{j=1}^{n} b_j = 123 - 115 = 8$$

$$b_4 = 8 \qquad \text{we have} \quad D_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

	D_1	D_2	D_3	D_4	a_i'
L_1	2	2	0 $\boxed{35}$	0 $\boxed{2}$	37
L_2	4 $\boxed{23}$	1 $\boxed{25}$	0	0 $\boxed{6}$	54
L_3	1 $\boxed{32}$	3	0	0	32
b_j	55	25	35	8	123

$$\min F_1(x) = 0 \cdot 35 + 0 \cdot 2 + 4 \cdot 23 + 1 \cdot 25 + 0 \cdot 6 + 1 \cdot 32 = 149$$

	D_1	D_2	D_3	D_4	a_i'
L_1	2 **-2**	2 **1**	0 $\boxed{35}$	0 $\boxed{2}$	37
L_2	4 $\boxed{23}$	1 $\boxed{25}$	0 **0**	0 $\boxed{6}$	54
L_3	1 $\boxed{32}$	3 **5**	0 **3**	0 **3**	32
b_j	55	25	35	8	123

Determination of indicator d_{ij} for all free fields $x_{ij} = 0$

$d_{ij} = c_{ij} - z_{ij}$, $d_{11} = -2$, $d_{12} = 1$, $d_{23} = 0$, $d_{32} = 5$, $d_{33} = 3$, $d_{34} = 3$

	D_1	D_2	D_3	D_4	a_i'
L_1	2 $\boxed{2}$	2 **3**	0 $\boxed{35}$	0 **2**	37
L_2	4 $\boxed{21}$	1 $\boxed{25}$	0 **-2**	0 $\boxed{8}$	54
L_3	1 $\boxed{32}$	3 **5**	0 **1**	0 **3**	32
b_j	55	25	35	8	123

$$\min F_2(x) = 2 \cdot 2 + 0 \cdot 35 + 4 \cdot 21 + 1 \cdot 25 + 0 \cdot 8 + 1 \cdot 32 = 145$$

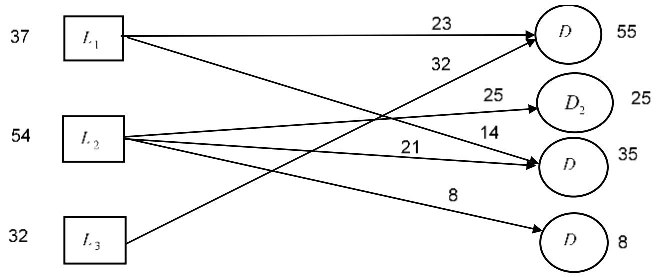
Again, found the values of indicators for all free areas $x_{ij} = 0$, and apropos we conclude that indicator $d_{23} = -2$, so the procedure have to continue for finding the optimal solution.

	D_1	D_2	D_3	D_4	a_i'
L_1	2 $\boxed{23}$	2 **1**	0 $\boxed{14}$	0 **0**	**37**
L_2	4 **3**	1 $\boxed{25}$	0 $\boxed{21}$	0 $\boxed{8}$	**54**
L_3	1 $\boxed{32}$	3 **3**	0 **1**	0 **1**	**32**
b_j	**55**	**25**	**35**	**8**	**123**

As is seen all calculated indicators are positive value then can be concluded that the solution is optimal, but for the indicator $d_{14} = 0$ can be found the other alternative optimal solution.

$$\min F_3(x) = 2\cdot 23 + 0\cdot 14 + 1\cdot 25 + 0\cdot 21 + 0\cdot 8 + 1\cdot 32 = 103$$

Optimal transport plan will appear as follows:

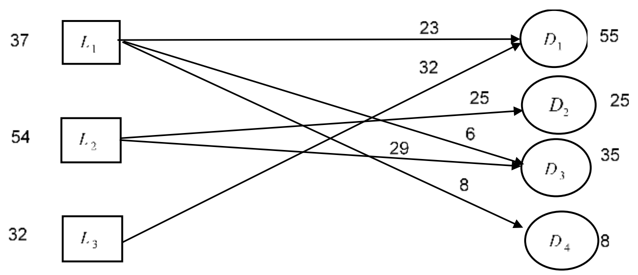


For indicator $d_{14} = 0$ found alternate optimal transportation plan

	D_1	D_2	D_3	D_4	a_i'
L_1	2 $\boxed{23}$	2	0 $\boxed{6}$	0 $\boxed{8}$	**37**
L_2	4	1 $\boxed{25}$	0 $\boxed{29}$	0	**54**
L_3	1 $\boxed{32}$	3	0	0	**32**
b_j	**55**	**25**	**35**	**8**	**123**

$$\min F_4(x) = 2\cdot 23 + 0\cdot 6 + 0\cdot 8 + 1\cdot 25 + 0\cdot 29 + 1\cdot 32 = 103$$

Alternate optimal transportation plan will appears as follows:



Finally most optimal solution is reached.

$$F_1(x) > F_2(x) > F_3(x) = F_4(x) \text{ or } 149>145>103=103$$

II. Conclusions

The optimal solution is obtained by following the rules and procedures for the initial and optimal methods. But, the problem of transport (optimal logistic cost) is much more complicated and difficult to be solved if the number of supply and demand stations is bigger. Of course, again there are solutions for these cases, but this requires more work and time.

Summary of procedures:

[1]. We must be sure that the offer is equal to demand, as needed we have to build delivery/destination ceters with zero transport cost per unit

[2]. The initial solution should be solvet in the beginning of the process;

[3]. The conclusion for the type of solution should be derived and if the solution degenerates then the process is followed by the application of the methods to eliminate degeneration;

[4]. If the indicators calculated for the empty fields are smaller than zero, the program of transport must be improved; Steps [3] and [4] have to be repeated until all the indicators are greater or equal to zero, then the optimal solution is reached.

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