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Abstract

In this paper, we have studied the impact of electron plasma frequency on Jeans instability of radiative magnetized quantum plasma. The analysis is carried out within the framework of the normal mode analysis technique which is modified due to the contribution of electron plasma frequency. The dispersion relation is reduced for both longitudinal and transverse modes of propagation. The condition of Jeans instability is modified due to the presence of magnetic field and electron plasma frequency in the transverse mode of propagation. From the curve, it is clear that electron plasma frequency has destabilizing influence in the system but the presence of a quantum parameter is reduced the destabilizing effect of electron plasma frequency and stabilized the system. The research will help to understand the stellar evolution in astronomical plasma.

Keyword – Electron plasma frequency, Quantum correction, Radiative heat-loss function, electrical resistivity, and magnetic field.

1. Introduction

The Jeans instability is the fundamental keywords to understand the star formation process which was discovered by Jeans [1] its play a crucial role in formation of astrophysical objects, i.e., stars are formed when dispersed matter in gaseous form starts coalescing together under the force of gravity continuously tries to shrink the objects and in this process, the density and temperature inside the stars rise. The time of billion years is required for the formation of heavenly objects while Jeans instability predicts it is a relatively faster process. According to Jean's criterion, an infinite homogeneous self-gravitating atmosphere is unstable for all wave numbers k less than Jeans' wave number $k_j = \left(\frac{G\rho}{S}\right)$ where ρ is the density, S is the velocity of sound in the gas, and G is the gravitational constant. This problem has been studied by several authors under varying assumptions of hydrodynamics and hydromagnetics, and a comprehensive account of these investigations has been given by Chandrasekhar [2] in his monograph on problems of hydrodynamics and hydromagnetics stabilities. He found that Jeans' criterion remains unaffected by the separate or simultaneous presence of rotation and magnetic field. In this direction, the thermal instability in cooling and expanding medium including self-gravity and conduction in the neutral fluid dynamics has been investigated by Gomez-Pelaez and Moreno-Insertis [3]. Nipoti and Posti [4] have studied the thermal stability of rotating optically thin plasma in a weakly magnetic field. The stability properties of thermal modes in cool prominence plasmas are investigated by Soler et al. [5]. Hobbs et al. [6] have investigated thermal instability in a cooling galactic corona fueling star formation in a galactic disk. Inoue and Omukai [7] have carried out the problem of thermal instability and the multiphase interstellar medium in the first galaxies.

.It is well-known, the quantum effect plays an important role in structure formation through the gravitational collapsing process of astrophysical objects. Pines [8] first introduced quantum plasma, he studied that the at very low temperature, the de Broglie wavelength $\lambda_B = \frac{h}{\sqrt{2m_e,\ K_bT}}$ (where $m_e,\ K_b, T$ are electron and ion masses, Boltzmann constant, and temperature) of electrons and ions is of the order of the dimension of the system, such as Debye length and Larmor radius. In this type of dense plasma system, the wave function is associated with particles overlap thus the plasma behaves like Fermi gas, and we would treat it as quantum plasmas. The quantum plasma system can be described by three well-known models, the Wigner-Poisson (WP) model, the Hartree model, and the Schrodinger-Poisson (SP) model or quantum hydrodynamic (QHD) model. A quantum multi-stream model for one and two-stream plasma instabilities are presented by Hass [9] and also investigated the stationary states of the nonlinear Schrodinger-Poisson model. Masood et al. [10] investigated the self-gravitational instability of a multi-component quantum plasma using Bohm potential and statistical terms on electrons and ions. In this way, many authors [11-14] have discussed Jean's instability including the different parameters in their research. Recently Sharma et al. [15] have analyzed the effect of electron inertia on radiative instability of optically thick plasma. Kumar et al. discussed the influence of the effect of photoelectron current on jeans instability of rotating quantum dusty plasma[16].

Thus, we find that a large number of studies are done for the quantum magnetohydrodynamic model (QMHD) with different parameters under various assumptions. But no one considers the quantum magnetohydrodynamic model with resistivity and electron inertia effect.

2. Equation of the problem

In this piece of work, we consider an infinite self-gravitating homogeneous plasma medium, which comprises electrons and single charged ions with electrical resistivity. The plasma is immersed in an ambient uniform magnetic field $\vec{H}(0,0,\mathrm{H})$ is in the z-direction. The basic radiative QMHD set of equations follows,

$$\frac{\partial \vec{V}}{\partial t} = -\frac{\nabla \delta p}{\rho} + \nabla \delta U + \frac{1}{4\pi\rho}\left(\nabla \times \vec{h}\right) \times \vec{H} + \frac{\hbar^2}{4m_e m_i}\nabla \frac{(\nabla^2 \delta\rho)}{\rho} \tag{1}$$

$$\frac{\partial \delta\rho}{\partial t} = -\rho\nabla.\vec{V} \tag{2}$$

$$\nabla^2 \partial U = -4\pi G\delta\rho \tag{3}$$

$$\frac{1}{(\gamma-1)}\frac{\partial \delta\rho}{\partial t} - \frac{\gamma}{(\gamma-1)}\frac{p}{\rho}\frac{\partial\delta\rho}{\partial t} + \rho\left(\mathcal{L}_\rho \delta\rho + \mathcal{L}_T \delta T\right) = \lambda \nabla^2 \delta T \tag{4}$$

$$\frac{\delta P}{P} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \tag{5}$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{H}\right) + \eta \nabla^2 \vec{h} + \frac{c^2}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2 \vec{h} \tag{6}$$

$$\nabla.\vec{h} = 0 \tag{7}$$

The above equations (1)-(7) represent momentum transfer equation, continuity equation, Poisson's equation, heat equation for a perfect gas and state equation, idealized Ohm's law with electron plasma frequency and resistivity, Gauss's law respectively.

Where, $\vec{V}(V_x, V_y, V_z)$, is the fluid velocity, δp is the fluid pressure, U gravitational potential, $\vec{h}(h_x, h_y, h_z,)$ is the magnetic field, $\delta\rho$ is the fluid density, G gravitational constant, γ is the ratio of two specific heat, δT is the temperature, λ is the thermal conductivity, $\mathcal{L}_\rho$ is the partial derivatives of the density dependent $(\partial\mathcal{L}/\partial T)_T$ heat-loss function, $\mathcal{L}_T$ is the partial derivatives of the temperature dependent $(\partial\mathcal{L}/\partial T)_\rho$ heat-loss functions, δT is the temperature, $\hbar$ Plank's constant divided by $2\pi, m_e$ and m_i are the electron and ion mass, and ω_{pe} is the electron plasma frequency.

Combining equation (4) and (5), we obtain the expression for δp as

$$\delta p = \left(\frac{\alpha + \sigma C^2}{\sigma + \beta}\right)\delta\rho \tag{8}$$

Where $\sigma = i\omega$ is the growth rate of the perturbation, and $C = \left(\frac{\gamma p}{\rho}\right)^{1/2}$ is the adiabatic velocity of sound in the medium, $s = \delta\rho/\rho$ is the condensation of the medium. The parameter $\alpha\ and\ \beta$ are

$$\alpha = (\gamma - 1)\left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho}\right)\ and\ \beta = (\gamma - 1)\left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p}\right)$$

We assume that all the perturbed quantities vary as

$$exp\{i(k_x x + k_z z + \omega t)\} \tag{9}$$

Where ω is the frequency of harmonic disturbances, k_x and k_z are the wave numbers in perpendicular and parallel direction to the magnetic field, respectively, such that $k_x^2 + k_z^2 = k^2$

Using equation (1) - (9) we obtain the following matrix relation.

$$X_{ij} Y_j = 0,\ \ i,j = 1,2,3,4, \tag{10}$$

Where X_{ij} is a 4×4 matrix whose elements are,

$$X_{11} = \left(\sigma + \frac{k^2 V^2}{A_1}\right), \quad X_{12} = 0 \quad X_{13} = 0, \quad X_{14} = \frac{ik_x}{k^2}\left(\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right)$$

$$X_{21} = 0, \quad X_{22} = \left(\sigma + \frac{k_z^2 V^2}{A_1}\right), \quad X_{23} = 0, \quad X_{24} = 0,$$

$$X_{31} = 0, \quad X_{32} = 0, \quad X_{33} = \sigma\,, \quad X_{34} = \frac{ik_z}{k^2}\left(\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right),$$

$$X_{41} = \frac{ik_x k^2 V^2}{A_1}, \quad X_{42} = 0, \quad X_{43} = 0, \quad X_{44} = -\left(\sigma^2 + \Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right)$$

Where $V = \frac{H}{(4\pi\rho)^{1/2}}$ is the Alfven velocity, $C' = \left(\frac{p}{\rho}\right)^{1/2}$ are isothermal velocities of sound, respectively. Also we have assumed the following substitution.

$$\Omega_J^2 = (k^2 C^2 - 4\pi G\rho), \quad \Omega_I^2 = (k^2\alpha - 4\pi G\rho\beta),$$

$$\Omega_T^2 = \left(\frac{\sigma\Omega_J^2 + \Omega_I^2}{\sigma + \beta}\right), \quad \Omega_m = \eta k^2, A_1 = (\sigma f + \Omega_\mathrm{m}), f = \left(1 + \frac{c^2 k^2}{\omega_{pe}^2}\right),$$

The general dispersion relation can be obtained from the determinant of the matrix of equation (10) gives the general dispersion relation as

$$\sigma\left(\sigma + \frac{k^2 V^2}{A_1}\right)\left(\sigma + \frac{k_z^2 V^2}{A_1}\right)\left(\sigma^2 + \Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right) - \sigma\frac{k_x^2}{k^2}\left(\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right)\left(\frac{k^2 V^2}{A_1}\right)\left(\sigma + \frac{k_z^2 V^2}{A_1}\right) = 0 \tag{11}$$

The dispersion relation (11) shows the combined influence of electron inertia, electrical resistivity, quantum correction, radiative heat-loss function, thermal conductivity and gravitating mode of the system. If we neglect the effect of electron inertia, resistivity, quantum correction and strength of the magnetic field, then dispersion relation (11) is similar to Ibanez [11].

3. Discussion

For the detailed investigation of the influence of electron inertia and electrical resistivity including the radiative magnetized quantum plasma, the dispersion relation is discussed for parallel and perpendicular propagations.

3.1 Parallel propagation

In this parallel propagation, we assume all the perturbations are longitudinal to the direction of the magnetic field ($k_x = 0,\ \ k_z = k$). Thus, the dispersion relation (11) can be simplified as

$$\sigma\left(\sigma+\frac{k^2V^2}{A_1}\right)^2\left(\sigma^2+\Omega_T^2+\frac{\hbar^2k^4}{4m_em_i}\right)=0 \tag{12}$$

The equation (12) shows the combined effect of thermal conductivity, magnetic field, self- gravitation, quantum plasma, electrical resistivity, heat loss function, and electron inertia, the above equation has a tree independent factors, each represents the different parameters. The first factor of equation (12) is $\sigma = 0$ and represents the natural stability of the system. The second factor of equation (12) gives the second-order equation as,

$$\sigma^2 f+\sigma\,\Omega_{\mathrm{m}}+k^2V^2=0 \tag{13}$$

The dispersion relation (13) is affected by the presence of electron inertia, resistivity, and magnetic field but is dispersion relation does not effect by heat loss function and quantum correction. Now the third factor of the equation of (12) is equating to zero and solved it we get the dispersion relation.

$$\sigma^3+\sigma^2\beta+\sigma\left(\Omega_j^2+\frac{\hbar^2k^4}{4m_em_i}\right)+\Omega_I^2+\beta\frac{\hbar^2k^4}{4m_em_i}=0 \tag{14}$$

The dispersion relation (14) is modified by thermal conductivity, quantum correction, and radiative heat loss function of the medium. This mode does depend not depend on the electron inertia, resistivity, and magnetic field. The condition of Jeans instability is obtained by a constant term of dispersion relation (14) is given as

$$(\gamma-1)\left(\mathcal{L}_TT-\mathcal{L}_\rho\rho+\frac{\lambda k^2T}{\rho}\right)+\frac{\hbar^2k^4}{4m_em_i}\left(\frac{\mathcal{L}_TT\rho}{p}+\frac{\lambda k^2T}{p}\right)<\frac{4\pi G\rho}{k^2} \tag{15}$$

The equation (15) represents a modified condition of Jeans instability due to the quantum correction but is independent that electrical resistivity, magnetic field and electron inertia in the longitudinal mode of propagation.

3.2 Perpendicular propagation

For this case, we assume all the perturbation are propagating perpendicular to the direction of the magnetic field, we take $k_x = k,\ \ k_z = 0$. The dispersion relation (11) can be written as

$$\begin{aligned}\sigma^4f+\sigma^3(f\beta+\Omega_{\mathrm{m}})+\sigma^2\left(f\Omega_j^2+f\frac{\hbar^2k^4}{4m_em_i}+k^2v^2+\beta\Omega_{\mathrm{m}}\right)\\+\sigma\left(f\Omega_I^2+\Omega_{\mathrm{m}}\Omega_j^2+\frac{f\beta\hbar^2k^4}{4m_em_i}+\frac{\Omega_{\mathrm{m}}\hbar^2k^4}{4m_em_i}+\beta k^2v^2\right)+\Omega_I^2\Omega_{\mathrm{m}}+\Omega_{\mathrm{m}}\beta\frac{\hbar^2k^4}{4m_em_i}\\=0\end{aligned} \tag{16}$$

The dispersion relation (16) shows the combined influence of the magnetic field, electron inertia, quantum correction, thermal conductivity, radiative heat-loss functions, and gravitating mode of the system. Thus equation (16) represents a gravitating Alfven mode modified by quantum correction, electron inertia, thermal conductivity, and radiative heat-loss functions. In the absence of quantum correction, radiative heat-loss function and thermal conductivity effect, this dispersion relation (16) are similar to obtained by Uberoi [12] excluding rotation effect in that case. If we ignore the quantum correction effect, then (16) reduces to that of Bora and Talwar [13]

The condition of instability is obtained from the constant term of the dispersion relation (16), and it is given by

$$\left[\eta k^2(\gamma-1)\left\{\left(\mathcal{L}_T T-\mathcal{L}_\rho \rho+\frac{\lambda k^2 T}{\rho}\right)+\frac{\hbar^2 k^4}{4m_e m_i}\left(\frac{\mathcal{L}_T T\rho}{p}+\frac{\lambda k^2 T}{p}\right)\right\}<\frac{4\pi G\rho}{k^2}\right] \tag{17}$$

We write the dispersion relation (16) in non-dimensional form, for showing the effects of different parameter on growth rate of instability, as

$$\sigma^{*4}f+\sigma^{*3}(f\beta^*+\eta^*k^{*2})+\sigma^{*2}\{f(k^{*2}-1)+fQ^*k^{*2}+k^{*2}V^{*2}+\beta^*\eta^*k^{*2}\}$$
$$+\sigma^*\{f(k^{*2}\alpha^*-\beta^*)+\eta^*k^{*2}(k^{*2}-1)+f\beta^*Q^*k^{*2}+\eta^*k^{*4}Q^*+\beta^*k^{*2}V^{*2}\}$$
$$+\eta^*k^{*2}(k^{*2}\alpha^*-\beta^*)+\beta^*\eta^*k^{*4}Q^*=0 \tag{18}$$

Where the various non-dimensional parameters are defined as

$$\sigma^*=\frac{\sigma}{\sqrt{4\pi G\rho}},\ k^*=\frac{kC}{\sqrt{4\pi G\rho}},Q^*=\frac{\hbar^2k_j^2}{4m_em_i},V^*=\frac{V\sqrt{4\pi G\rho}}{C},\lambda^*=\frac{(\gamma-1)T\lambda\sqrt{4\pi G\rho}}{\rho C^2},\ \mathcal{L}_\rho^*=\frac{(\gamma-1)_\rho\mathcal{L}_\rho}{C^2\sqrt{4\pi G\rho}},\ \mathcal{L}_T^*=\frac{(\gamma-1)_\rho T\mathcal{L}_T}{\rho\sqrt{4\pi G\rho}},\ \alpha^*=\left(\frac{1}{\gamma}(\mathcal{L}_T^*+\lambda^*k^{*2})-\mathcal{L}_\rho^*\right),\ \beta^*=(\mathcal{L}_T^*+\lambda^*k^{*2}),\ \eta^*=\frac{\eta\sqrt{4\pi G\rho}}{C^2},\Omega_I^{*2}=(k^{*2}\alpha^*-\beta^*),$$
$$\Omega_j^{*2}=(k^{*2}-1) \tag{19}$$

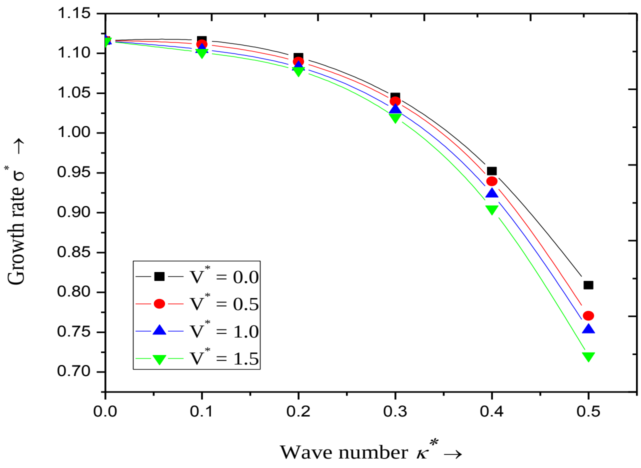

Fig.1 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the magnetic field $V^*=(0,0.5,1,1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^*=\eta^*=\mathcal{L}_T^*=\lambda^*=Q^*=0.5\ and\ f=0$

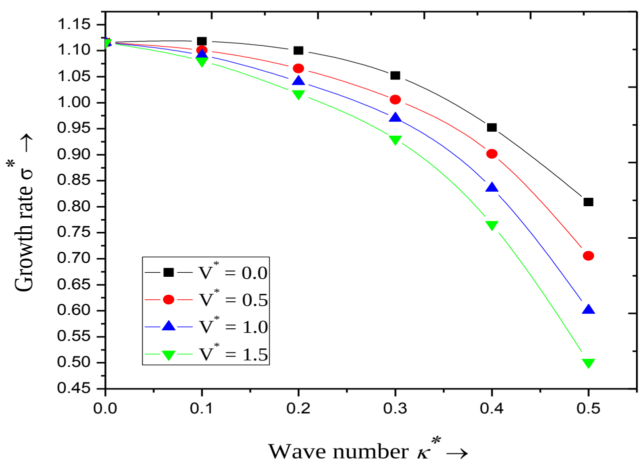

Fig.2 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the magnetic field $V^* = (0, 0.5, 1, 1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^* = \eta^* = \mathcal{L}_T^* = \lambda^* = Q^* = 0.5$ *and* $f = 0.5$

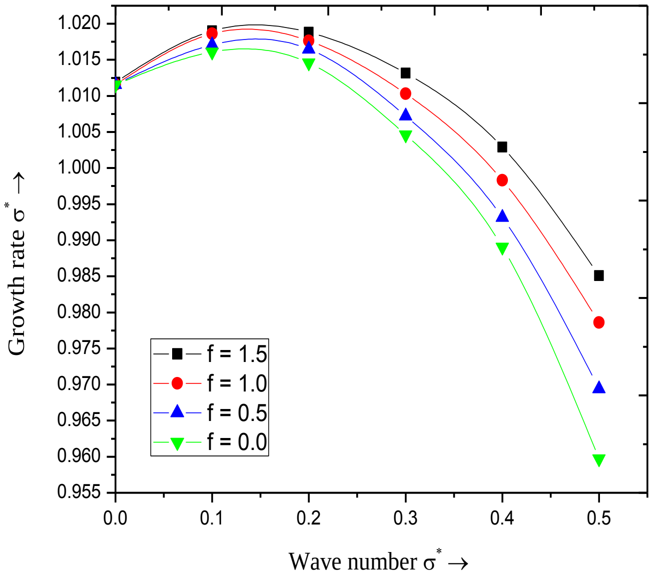

Fig.3 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the Electron inertia f $= (0, 0.5, 1, 1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^* = \mathcal{L}_T^* = \eta^* = \lambda^* = V^* = 0.5$ *and* $Q^* = 0$

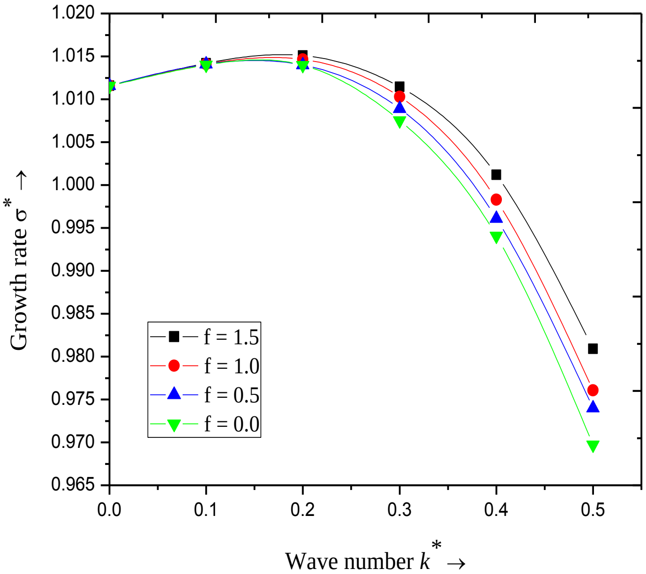

Fig.4 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the Electron inertia f $= (0, 0.5, 1, 1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^* = \mathcal{L}_T^* = \eta^* = \lambda^* = Q^* = 0.5$ *and* $Q^* = 0.5$

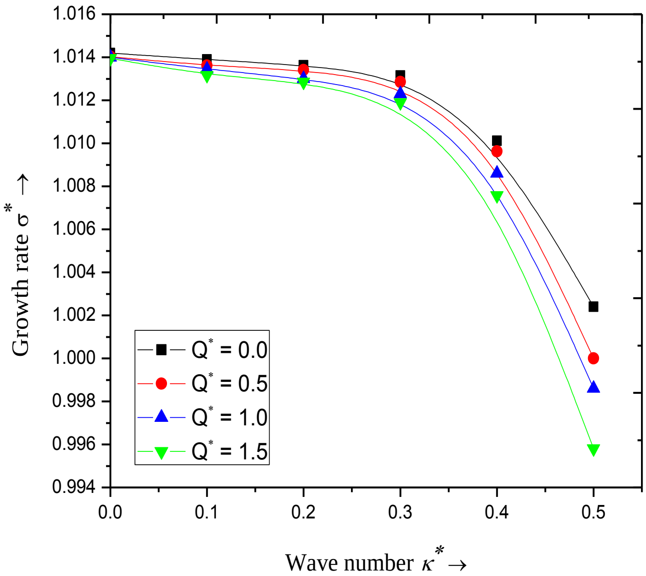

Fig.5 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the quantum correction $Q^* = (0, 0.5, 1, 1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^* = \mathcal{L}_T^* = \lambda^* = \eta^* = V^* = 0.5\ and\ f = 0$

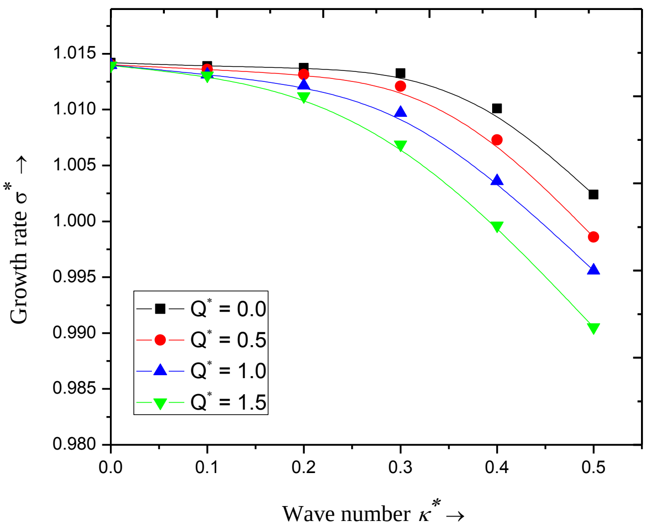

Fig.6 The growth rate (σ^*) is plotted against the non-dimensional wave number (k^*) with variation in the quantum correction $Q^* = (0, 0.5, 1, 1.5)$, keeping the values of other parameters are fixed, as $\mathcal{L}_\rho^* = \mathcal{L}_T^* = \lambda^* = \eta^* = V^* = 0.5\ and\ f = 0.5$

From the curves, we find that in figure 1-2, the magnetic field has a stabilizing impact in the system but the presence of electron inertia the system is unstable. In figure 3-4 we conclude that the electron inertia has a destabilizing effect but in the presence of quantum parameter the destabilizing effect is reduced. In the curve 5-6, we see that the quantum parameter have to stabilize effect but in the presence of electron inertia the system is destabilized so that the quantum parameter tries to decrease the destabilizing effect

4. Conclusion

The present problem has been analyzed in the framework of the quantum fluid theory. We have derived a general dispersion relation using normal mode analysis and QMHD equation. In the case of parallel propagation, the Jeans instability is modified in the presence of radiation and quantum correction but is independent of the electron inertia and electrical resistivity. The dispersion relation for the perpendicular mode is affected by all the parameters. The gravitational instability is obtained which is modified by electrical resistivity, thermal conductivity, radiative heat-loss function, and quantum correction. From the curves, we find that the magnetic field and quantum correction have to stabilize effect but electron inertia has a destabilizing effect on the growth rate of instability.

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