Distributed Voltage Control in Distribution Network with Area Partition

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***Abstract*—The deployment of distributed generations(DGs) poses increasing burden to sensing and communication in centralized voltage control. To address this issue, a distributed voltage control method is proposed in this paper. In the proposed method, the whole distribution network is separated into several areas and each area is equipped with a sub-controller. The idea of alternating direction method of multipliers(ADMM) is used for the distributed voltage control design by exchanging information of boundary variables between two adjacent areas. In this way, the requirements for data collection and communication is reduced. Case studies on a modified IEEE 33 bus system with three area partition demonstrate the performance of the proposed distributed voltage control method.**

***Index Terms*—Voltage control, distribution generation, distribution network, distributed control, ADMM**

I. Introduction

Voltage/Var control(VVC) is the fundamental requirements to ensure the security of distribution systems operation. The main tasks of VVC in distribution systems is to maintain the distribution network(DN) feeder voltages within an allowed range, typically ±5% to its nominal value [1]. Conventionally, VVC is achieved by utility-owned equipments, such as transformers with on-load tap changer(OLTC), step voltage regulators(SVRs), and shunt capacitor banks(CBs). In recent decades, the increasing deployment of DGs, such as photovoltaic(PV) panels and wind generation, results in larger voltage fluctuation at a faster rate that the conventional VVC devices may fail to handle. Therefore, DGs with power electronics interface are encouraged to provide fast reactive power support to mitigate the disturbance to voltages [2].

VVC methods can be classified into three categories: local, centralized and distributed(or termed as decentralized) [3]. Local control is simple and easy to implement, however, it usually suffers from degraded performance and stability issue as only relying on local measurements [4]. In centralized control, the control controller gathers all necessary information, such as load, generation and system parameters, to perform a central computation, then send the control command to VVC devices. In this way, the global optimum is guaranteed in centralized methods. Centralized methods highly dependent on complex communication and computation, making it impractical to large distribution systems and vulnerable to potential imperfect communication infrastructure. Recently, particular attention has been devoted to distributed control methods. Rather than gathering all the information and parameters, distributed control methods generate control command by many local agents via information exchange between neighbouring agents [5]. In this way, the requirements on data collection and communication are greatly reduced. Rather than directly solving a large-scale optimization problem in centralized methods, the large-scale main problem are decomposed into many small-scale sub-problems which are solved and coordinated by local agents in distributed methods. Hence the distributed ones are superior in terms of calculation speed and maximum problem size that can be addressed.

Based on augmented Lagrangian decomposition, the alternating direction method of multipliers(ADMM) has been successfully applied in the area of voltage control [6]–[9]. Ref. [6] adopted ADMM on optimal power flow(OPF) problem and tested its performance on a real-life 2065-bus system. The authors in [7] explored distributed approaches to control PV inverters reactive power output and the idea is extended into unbalanced distribution systems in [8]. In reference [9], an ADMM-based fully distributed reactive power optimization algorithm was proposed by dividing the distribution networks

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into several areas.

However, the above studies only considers reactive power control from DGs, their coordination with conventional VVC devices of discrete nature is not studied. In fact, ADMM is original developed to solve problems with continuous variables, thus cannot be directly applied to solve mixed-integer OPF problem. To address this issue, this paper presents a distributed voltage control method(D-VVC), in which an improved ADMM in [10] is adopted to tackle with the mixed-integer problems. The basic idea is to include new variables in the objective function, and to update the discrete variables at each iteration step according to auxiliary equality constraints. Compared with previous studies, the main contributions of this paper can be summarized as follows:

- An ADMM-based D-VVC is proposed, the whole DN is divided into several areas, each area only needs to communicate boundary variables with neighbouring areas. Thus, a centralized control centre is not required.
- The proposed method not only takes advantage of the rapid response inverter-based DGs, but also of the conventional VVC devices with discrete characteristics.
- The proposed method can regulate the voltage profile within allowed range while minimizing the network power losses. The performance of the proposed method is demonstrated by case studies.

The remainder of this paper is organized as follows. Section II introduces system model and problem formulation. Section III presents the details of proposed D-VVC. The effectiveness of the proposed framework is validated in section IV while section V conclusions this paper.

II. System Modelling and Problem Formulation

A. Model of distribution network

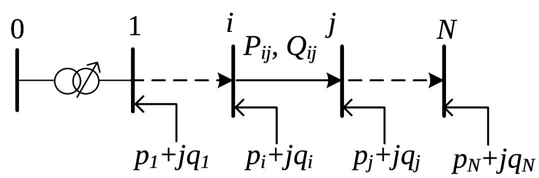

Fig. 1. Topology of a typical radial DN

Typically a distribution network is operated at radial topology, as shown in Fig. 1. Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ represents graph of DN, where $\mathcal{N} := \{0, 1, \ldots, N\}$ denotes the set of buses and $\mathcal{L} := \{1, 2, \ldots, N\}$ denotes the line set. The power flow of DN can be described by Distflow equation [11]

$$P_{ij} = \sum_{k \in \mathcal{N}_j} P_{jk} - p_j + r_{ij}\ell_{ij} \tag{1a}$$

$$Q_{ij} = \sum_{k \in \mathcal{N}_j} Q_{jk} - q_j + x_{ij}\ell_{ij} \tag{1b}$$

$$v_i - v_j = 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right) - \left(r_{ij}^2 + x_{ij}^2\right)\ell_{ij} \tag{1c}$$

$$\ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \tag{1d}$$

where $v_i := |V_i|^2$ represents the squared voltage magnitude. Assuming the loss is negligible compared with line flow and DN is under relatively flat voltage profile during normal operation, the Distflow equation can be approximated to linear model, such approximation introduces a modelling error at about 0.25%(1%) if there is a 5%(10%) deviation in voltage magnitude [12]. The linear model can be given as,

$$P_{ij} = \sum_{k \in \mathcal{N}_j} P_{jk} - p_j \tag{2a}$$

$$Q_{ij} = \sum_{k \in \mathcal{N}_j} Q_{jk} - q_j \tag{2b}$$

$$V_i - V_j = r_{ij}P_{ij} + x_{ij}Q_{ij} \tag{2c}$$

$$p_j = p_j^g - p_j^l \tag{2d}$$

$$q_j = q_j^g + q_j^c - q_j^l \tag{2e}$$

For OLTC transformer, the step changer is assumed on the secondary side and can be described by the following model,

$$V_1 = V_0\left(1 + n_{tap}\right)\Delta tap \tag{2f}$$

We assume that load real/reactive power consumption is uncontrollable, DG active power output is also uncontrolled as they operate at maximum power point tracking(MPPT) mode to fully capture renewable energy. So voltage regulation is accomplished by adjusting DG reactive power output as well as CB and OLTC transformer tap positions.

B. Formulation of VVC problem

The main purpose of VVC is to regulate the DN feeder voltages into an allowed range by adjusting the node reactive power injection and OLTC tap position. Note that such adjustment has a strong influence on the network losses [13]. So the optimization model of VVC is to minimize the network losses while maintaining the voltage profile in the allowed range. The VVC model can be expresses as,

$$\{\boldsymbol{q}^*, \boldsymbol{n}^*\} := \underset{\boldsymbol{q},\boldsymbol{n}}{\arg\min} f\left(\boldsymbol{q}, \boldsymbol{n}\right) = \sum_{(i,j) \in \mathcal{L}} r_{ij} \cdot \frac{P_{ij}^2 + Q_{ij}^2}{v_0} \tag{3a}$$

subject to

$$\underline{\boldsymbol{V}} \leqslant \boldsymbol{V} \leqslant \overline{\boldsymbol{V}} \tag{3b}$$

$$-\sqrt{\boldsymbol{S^2} - \boldsymbol{p^2}} \leqslant \boldsymbol{q} \leqslant \sqrt{\boldsymbol{S^2} - \boldsymbol{p^2}} \tag{3c}$$

$$\underline{\boldsymbol{n}} \leqslant \boldsymbol{n} \leqslant \overline{\boldsymbol{n}} \tag{3d}$$

where $\overline{\boldsymbol{V}}$ and $\underline{\boldsymbol{V}}$ are the upper/lower limit of voltage magnitude respectively. $\boldsymbol{q}$ represents the DG reactive power output injection which is constrained by DG rated apparent power $\boldsymbol{S}$ and instantaneous active power $\boldsymbol{p}$. $\boldsymbol{n}$ denotes the discrete variable which represents the tap positions of OLTC transformer and CBs while $\overline{\boldsymbol{n}}$ and $\underline{\boldsymbol{n}}$ denote their upper/lower limit.

III. Proposed Distributed Voltage Control Strategy

The optimization model in (3a)-(3d) is in centralized design. In this section, we will explore its decomposition structure and solve the optimization problem in a distributed fashion. ADMM is adopted in this paper as it shows better numerical stability and convergence rate than other decomposition technique such as dual-ascent. Firstly, the improved ADMM with mixed-integer objectives is briefly described, then the ADMM-based D-VVC will be introduced.

A. Improved ADMM with mixed-integer variables

ADMM is originally applied in the scientific area such as machine learning and signal processing, in which area most problems are continuous value based. Thus ADMM cannot directly used to solver problems with integer variables. In this paper, an improved ADMM presented in [10] is adopted to solve the mix-integer VVC problem.

The optimization problem with mixed-integer variables is as follows,

$$minimize\ f(x,z) \tag{4}$$

with variable $x \in \mathbb{R}^n$, $z \in \mathcal{Z}^m$. Define a new inner function $g(\cdot,\cdot) : \mathbb{R}^n \times \mathcal{Z}^m \to \mathbb{R}^p$, then the optimization problem in (4) can be written as the following equivalent form,

$$minimize\ f(v) \tag{5a}$$

subject to

$$v = g(x,z) \tag{5b}$$

The augmented Lagrangian can be represented as,

$$L_\rho(x,v,z,u) = f(v) + \frac{\rho}{2}\|v - g(x,z) + u\|_2^2 - \frac{\rho}{2}\|u\|_2^2 \tag{6a}$$

where $\rho > 0$ is the augmented Lagrangian parameter.
The improved ADMM consists of the following iterations,

$$\left(x^{(k)}, v^{(k)}\right) = \underset{x\in\mathbb{R}^n, v\in\mathbb{R}^p}{arg\ min}\ L_\rho\left(x, v, z^{(k-1)}, u^{(k-1)}\right) \tag{6b}$$

$$z^{(k)} = arg\ \underset{z\in\mathcal{Z}^m}{min}\|v^{(k)} - g\left(x^{(k)}, z\right) + u^{(k-1)}\|_2^2 \tag{6c}$$

$$u^{(k)} = u^{(k-1)} + v^{(k)} - g\left(x^{(k)}, z^{(k)}\right) \tag{6d}$$

B. ADMM-based distributed voltage control

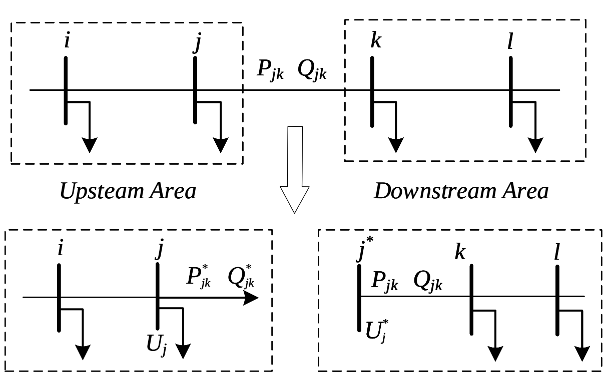

Fig. 2. Diagram of distributed VVC

In the proposed D-VVC, the whole DN is divided into K areas. Taking the DN with two area separation shown in Fig. 2 as an example, after network separation, the boundary node in the upstream will be copied to the downstream as a virtual slack bus, while the power flow in the boundary branch is viewed as a virtual load in the upstream. For an arbitrary area a, define the internal state variables of area a as $\boldsymbol{x}_{a,int} = [\boldsymbol{P}_i; \boldsymbol{Q}_i; \boldsymbol{U}_i; \boldsymbol{P}_{ij}; \boldsymbol{Q}_{ij}], i \in \mathcal{N}_a, (i,j) \in \mathcal{L}_a$, the boundary variables with adjacent area b are denoted as $\boldsymbol{x}_{ab} = \left[\boldsymbol{P}_{ij}^{ab}; \boldsymbol{Q}_{ij}^{ab}; \boldsymbol{U}^{ab}\right]$. Let $\mathcal{R}^a$ denote the neighbour set of area a. All the boundary variables of area a is collected into $\boldsymbol{x}_{a,adj} = \{\boldsymbol{x}_{ab} \mid \forall b \in \mathcal{R}^a\}$ and define $\boldsymbol{x}_a = [\boldsymbol{x}_{a,int}; \boldsymbol{x}_{a,adj}]$ as the state variables of area a. The augmented Lagrange function of problem (3) with Lagrangian parameter $\boldsymbol{\rho}$ is given as,

$$L(\boldsymbol{q}, \boldsymbol{n}, \boldsymbol{s}, \boldsymbol{\lambda}) = \sum_{a=1}^{K} L_a\left(f\left(g\left(\boldsymbol{q}, \boldsymbol{y}\right)\right), \boldsymbol{s}_a, \boldsymbol{\lambda}_a\right) \tag{7a}$$

for each area a,

$$\begin{aligned} L_a\left(\boldsymbol{q}_a, \boldsymbol{n}_a, \boldsymbol{s}_a, \boldsymbol{\lambda}_a\right) = f\left(g\left(\boldsymbol{q}_a, \boldsymbol{y}_a\right)\right) + \\ \sum_{b\in\mathcal{R}^a}\left[\boldsymbol{\lambda}_{ab}^\top\left(\boldsymbol{x}_{ab} - \boldsymbol{s}_{ab}\right) + \frac{\boldsymbol{\rho}}{2}\|\boldsymbol{x}_{ab} - \boldsymbol{s}_{ab}\|_2^2\right] \end{aligned} \tag{7b}$$

In the proposed distributed framework, the sub-problem optimization in each area is conducted in parallel, then the adjacent area will exchange the bound data and update the global value of boundary data locally. After that, a new round of optimization is performed until the boundary data converges. The iteration procedure is as follows,

$$\left\{\boldsymbol{q}_a^{k+1}, \boldsymbol{y}_a^{k+1}\right\} = argmin\ L_a\left(f\left(g\left(\boldsymbol{q}_a^k, \boldsymbol{y}_a^k\right)\right), \boldsymbol{s}_a^k, \boldsymbol{\lambda}_a^k\right) \tag{8a}$$

$$\boldsymbol{n}_a^{k+1} = argmin\|g\left(\boldsymbol{q}_a^{k+1}, \boldsymbol{n}_a\right) - g\left(\boldsymbol{q}_a^{k+1}, \boldsymbol{y}_a^{k+1}\right)\|_2^2 \tag{8b}$$

$$\boldsymbol{s}_{ab}^{k+1} = \frac{x_{ab}^{k+1} + x_{ba}^{k+1}}{2}, \forall b \in \mathcal{R}^a \tag{8c}$$

$$\boldsymbol{\lambda}_{ab}^{k+1} = \boldsymbol{\lambda}_{ab}^k + \boldsymbol{\rho}\left(\boldsymbol{x}_{ab}^{k+1} - \boldsymbol{s}_{ab}^{k+1}\right), \forall b \in \mathcal{R}^a \tag{8d}$$

where $\boldsymbol{\lambda}_a = \{\lambda_{ab} \mid b \in \mathcal{R}^a\}$ and $\boldsymbol{s}_a = \{\boldsymbol{s}_{ab} \mid b \in \mathcal{R}^a\}$ as the dual variables and global variables, respectively. The termination criterion is that the primal residual $\delta_{pri}^k = \left\{\left|\boldsymbol{x}_{a,adj}^k - \boldsymbol{s}_a^k\right| \forall a\right\}$ and dual residual $\delta_{dual}^k = \left\{\left|\boldsymbol{s}_a^k - \boldsymbol{s}_a^{k-1}\right| \forall a\right\}$ is within a predetermined tolerance $\delta = \left\| \begin{matrix} \delta_{pri}^k \\ \delta_{dual}^k \end{matrix} \right\|_\infty < \sigma$.

IV. Case Study

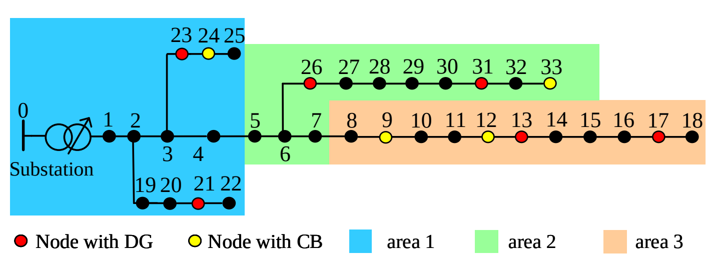

Fig. 3. Topology of Modified IEEE-33 bus system

To verify the effectiveness of the proposed D-VVC, case studies are performed on a modified IEEE-33 bus system, the network parameters can be found in [14]. The studies system is partitioned into three areas as shown in Fig. 3. There is no standard rule in determining the area partition in D-VVC, areas can be divided according to the need in real practice or by other advanced area partition techniques [15], [16]. The daily generation and load profile are obtained from the National Renewable Energy Laboratory (NREL) Renewable Resource Data Center [17], and the normalized generation and load are shown in Fig. 4 and Fig. 5, respectively. The DGs are operated at MPPT mode, each with capacity of 0.6MW. The OLTC transformer has a ±5% tap range with 20 tap positions and CBs are with 0.3MVar and 10 positions. Consider the different characteristics of different VVC devices, the conventional VVC devices are dispatched in a period of 2 hours to avoid frequent operation while DGs reactive power outputs are dispatched every 5 min to provide fast voltage support. The voltage upper/lower limits are set as 1.05 pu and 0.95 pu, respectively. Case studies are implemented on Matlab r2020a with Yalmip Toolbox [18] and the parallel sub-problems are solved by Gurobi solver [19].

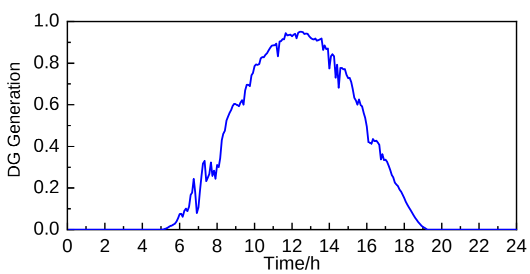

Fig. 4. DG generation

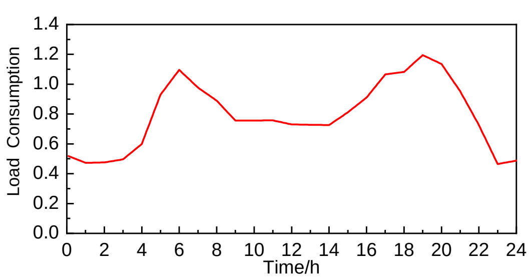

Fig. 5. Load profile

A. Convergence analysis

To perform convergence analysis, the proposed D-VVC is implemented with different Lagrangian parameter ρ. The tolerance is chosen as 10^{-7} and the residual in each iteration is recorded in Fig. 6. As shown in Fig. 6, the algorithm always converges within several iterations when ρ is in a certain range, demonstrating the good convergence ability of ADMM. Generally, the algorithm converges faster with a larger penalty parameter ρ, just as shown in this study. However, too large ρ may also make the algorithm converge slowly due to an oversize update per step, which is the case when ρ increases from 0.5 to 5.

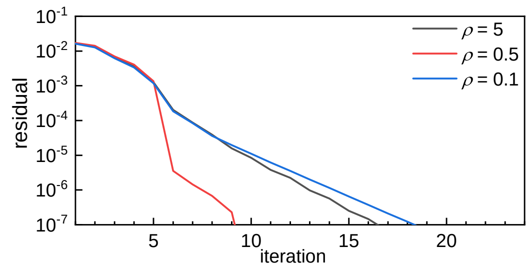

Fig. 6. Convergence with different ρ

B. Time-series simulation

In this section, a 24-hour time-series simulation is conducted and the performance of proposed D-VVC is compared with centralized VVC(C-VVC). The voltage profile in two simulations is shown in Fig. 7. In general, the bus voltages are relatively low during night time, and the overall network voltages become higher as DGs have more real power output. Some bus voltages reaches voltage upper limit during 10:00-14:00. In the whole 24 hour, the bus voltages maintain in allowed range both in D-VVC and C-VVC.

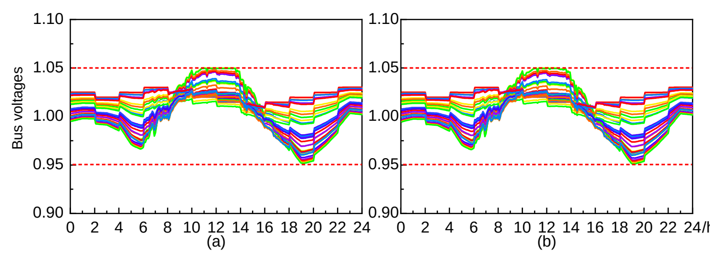

Fig. 7. Voltage profile (a). C-VVC (b). D-VVC

The DG reactive power outputs and operation of conventional VVC devices are shown in Fig. 8 and Fig. 9, respectively. As can be seen in Fig. 8, during most of the time, DG generate reactive power to support bus voltages, especially when there are low DG generation and high load demand. However, during 10:00-14:00 when DGs are in high real power outputs, some DGs will absorb reactive power to reduce the bus voltages. In Fig. 9, the actions of OLTC tap changer are the same in the two simulations. The operation of CBs at node 12, 24, 33 are the same in C-VVC and D-VVC, demonstrating the performance of the D-VVC in addressing discrete variables. Note that the operation of CB at node 9 is slightly different in two simulations. The reason for the difference is that when projecting the contiguous variables to the discrete sets, the obtained results may be different from directly solving the centralized problem. However, as can be seen in the later simulation, the distance of this difference is quite small to both the solution and the objective function of the VVC problem.

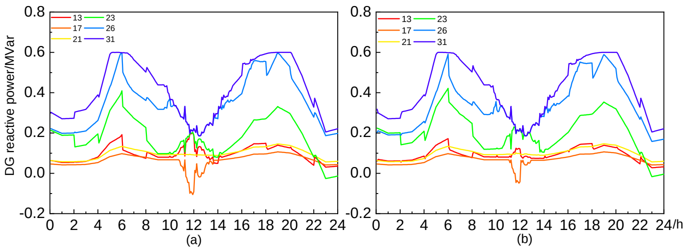

Fig. 8. DG reactive power output (a). C-VVC (b). D-VVC

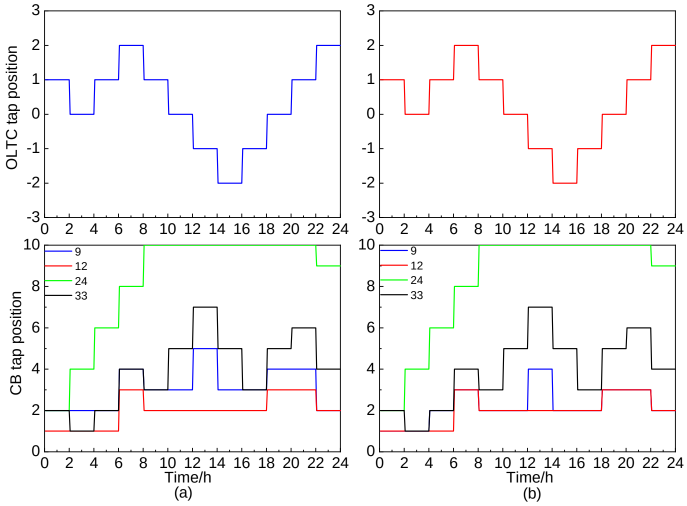

Fig. 9. Conventional VVC devices control action (a). C-VVC (b). D-VVC

To further demonstrate the accuracy of the D-VVC, the power losses in D-VVC and the power loss error with C-VVC are shown in Fig. 10 and Fig. 11, respectively. In Fig. 10, the network power losses increase due to DG units generate more reactive power to support the bus voltages. From Fig. 11 we can see that, power losses in D-VVC maintain almost the same level with the C-VVC in the 24 hour simulation, the maximum error is within 1.4% in this period.

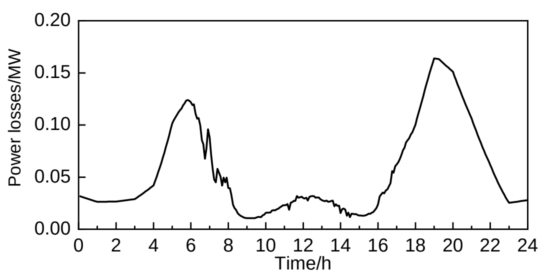

Fig. 10. Power losses

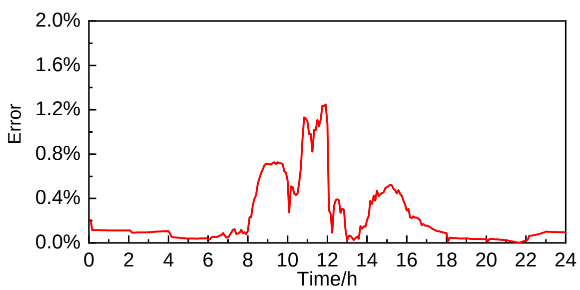

Fig. 11. Error of power losses with C-VVC

V. Conclusion

This paper presents an ADMM based distributed voltage control method in DN. Compared with previous distributed works, both the conventional VVC devices and the DGs with fast response speed are utilized in the proposed method. By adding equality constraints, the conventional VVC devices with discrete nature are addressed using a projection method. In comparison with C-VVC, the proposed D-VVC has less requirements on data collection and information communication. Case studies on a modified IEEE-33 bus system demonstrate the performance of the proposed distributed method.

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