An Extended Extreme Shock Maintenance Model for a Deteriorating System Under Monotone Process.

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AN EXTENDED EXTREME SHOCK MAINTENANCE MODEL FOR A DETERIORATING SYSTEM UNDER MONOTONE PROCESS.

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ABSTRACT

We investigate a failing system that experiences unpredictably harsh shocks from its surroundings. Assume that while the system is working, every shock will cause some damage to it; however, shocks that cause only minor damage are innocuous for the system, whereas shocks that cause significant damage may cause the system to fail. The system's disintegration is brought about by both the outside shocks and the interior load. An extended extreme shock (EES), the maintenance model for systems disintegrating under monotone processes by using increasing Alpha series processes (ASP), is studied. Perceptionally an exact expression for the long -run mean cost for each unit time under N strategy is derived and an optimal strategy N^* for reducing long -run mean cost for each unit time is determined. A numerical illustration is also given.

Keywords: Renewal Process, Alpha Series Process, Geometric Process, Geometric distribution, Substitution Strategy, Exponential distribution, Shock Model.

1 Introduction

The analysis of maintenance issues is still a crucial area of reliability. A large number of academics have produced a number of results in the field of "repair as new." The general premise of this research study on repair substitution problems is that a system that has been repaired is "as good as new." However, it is not always accurate or the same for a failing system. Due to wear and tear from use and accumulated damage, the majority of repairable systems disintegrate over time. The majority of maintenance models concentrate solely on the internal cause of system failure rather than considering an external cause. A system could malfunction as a result of outside factors like shock. However, shocks with "minor" levels of damage are not harmful to the system, whereas shocks with higher levels of damage could lead to have damage are not harmful to the system, whereas shocks with higher levels of damage could lead to system failure. Both internal and external loads influence how the system is determined. The amount of shock damage that a system can withstand from external causes reduces as more repairs are made, whereas for internal reasons, the time between repairs grows as more repairs are completed. For a system that is degrading, Chen and Li (2007) presented and investigated an EES maintenance model in which the subsequent repair time is a geometric process (GP).

The system is considered in two aspects: internal and external. First, if the system fails due to a shock, it is repaired or substituted with a new and identical one. Due to ageing and constant wear and tear, the repair time increases and tends to infinity, eventually the system becomes unrepairable, so the repair time cannot be ignored. Therefore, we model the repair times after system failures as an increasing ASP. It was introduced by Braun et al. (2005).

Although the GP is a better model for systems disintegrating, Braun et al. (2008) were created an alternative model called the ASP that has similar qualities. Additionally, Braun et al. (2008) emphasised that while the increasing GP only increases in time logarithmically, the decreasing geometric process is almost certain to undergo an explosion at some point. The ASP grows with time either exponentially. It was also claimed that a central limit theorem is satisfied by the ASP but not by the GP .They showed that the ASP and the increasing GP both have a finite first moment under certain generic conditions.

Next, shocks are studied from system surroundings. There were several papers that called extreme trauma models. In these models, a shock is called a fatal shock, if the amount of damage a shock does to the system exceeds a certain threshold and the system fails. These types of shock models are called “extreme shock models.” Practically speaking, the systems disintegrating after repair should be weak and easy to break. Consequently, as the number of repairs performed increases, the threshold value k for exceeding lethal shock decreases.

Below are some preliminary definitions and findings about the ASP.

Definition1.1. Let $\{X_K, k = 1,2,3, \dots \dots \dots . .\}$be a sequence of non-negative independent random variables (RV), if the distribution function of X_k is $F_k(x) = F(k^\alpha x)$ for α is real number, $k = 1,2,3 \dots \dots$and then$\{X_K, k = 1,2,3, \dots \dots \dots . .\}$is said to form an ASP. The real α is called the exponent of process. See e.g Braun (2005)

Remark1.1. Let $\{X_K, k = 1,2,3, \dots \dots \dots . .\}$ be a sequence of non-negative independent RV, if the density function of $X_k$is$f_k(x) = k^\alpha f(k^\alpha x)$.

Remark1.2. If $\{X_K, k = 1,2,3, \dots .\}$ is an ASP and $E(X_1) = \lambda$ then $E(X_k) = \frac{\lambda}{k^\alpha}$. See.Braun(2005)

Remark1.3. Wald’s equation. If $X_1, X_2, \dots \dots$independent and identically distributed (i.i.d) RV are having finite expectations and if $N$the stopping is time for $X_1, X_2, \dots \dots$ such that $E[N] < \infty$, then $E[\sum_{n=1}^{N} X_n] = E[N]E[X_1]$ See e.g. Lam (1988b)

2 Model Assumptions

Under the ensuring presumptions, we assume an EES the maintenance model for a systems disintegrating.

A1. Initially, a fresh system was established. When a system malfunctions, it is either repaired or substituted out for an entirely new one.

A2. When the system begins working, the shocks from the surroundings show up, as indicated by a renewal process. Let $\{W_{ki}, i = 1,2,3 \ldots..\}$ be the intervals between the$(i-1)^{st}$ and the i^{th} shock after the $(k-1)^{st}$ repair. Let $E(W_{11}) = \lambda$. Assume that $\{W_{ki}, i = 1,2,3 \ldots \ldots\}$ are i.i.d. sequences for all k.

A3. Let $\{D_{ki}, i = 1,2,3 \ldots.\}$ be the order of magnitude of shock damage caused by the i^{th} shock after the $(k-1)^{st}$ repair. Let $E(D_{11}) = \beta$. Expect that $\{D_{ki}, i = 1,2,3 \ldots.\}$ are i.i.d. sequences for all k.

A4. In the k^{th} working stage, after the $(k-1)^{st}$ repair, the system fails if the amount of shock damage initially exceeds $a^{k-1}M$ where $0 < a \leq 1$, the system fails, it is shut,and random shocks at repair time have no effect on the system.

A5. Allow Y_1 be the repair time after the 1^{st}failure, and$F(y)$ be the distribution functions of Y_1.For, $k = 1,2,3 \ldots$Let Y_k signify the repair time following the k^{th} failure. The distribution function of Y_k is then assumed to be $F(k^\alpha x)$ for $t \geq 0, \alpha > 0$ is a genuine number. This generates an ASP with increasing successive repair times $\{Y_k, k = 1,2,3 \ldots \ldots\}$, Also assume that $E(Y_1) = \mu \geq 0, \mu = 0$ where the repair time is negligible. $N_n(t)$is the process of counting the number of shocks after the $(k-1)$st repair. It is clear that$E(Y_k) = \frac{\mu}{k^\alpha}$.

A6. Allow Z to be the substitution time, with$E(Z) = \tau$.

A7. The process$\{D_{ki}, i = 1,2,3 \ldots.\}$,$\{W_{ki}, i = 1,2,3 \ldots..\}$, $\{Y_k, k = 1,2,3 \ldots \ldots\}$ and Z are independent.

A8. The maintenance cost rate is c, the reward rate is r and the substitution rate is R.

3 Average cost rate

Definition 3.1. In general, Let T_1 be the first substitution time, for $k \geq 2$.

Let T_n be the time between the $(k-1)^{st}$ and the k th substitution. Then obviously

$\{T_n, n = 1,2,..\}$ generates the renewal process.

The point of the review is to find an optimal substitution N with the end goal that the long run mean cost for every unit time is limited.

According to renewal reward hypothesis of Ross (1983), the long-run mean cost for each unit time under the substitution strategy N is given by

$$C(N) = \frac{The\ expected\ cost\ incurred\ in\ a\ cycle}{The\ expected\ length\ of\ a\ cycle} \tag{1}$$

In our models, we do not specify the distribution of the shocks inter arrivals, i.e., the distributions of $W_{ki}$and D_{ki} may be arbitrary, it uses the geometric distribution.

We want to initially work out the geometric distribution and the expectation for X_k, the genuine working time of the system after the $(k-1)^{st}$ repair.

Denote $$L_k = \min\{l: D_{ki} > a^{k-1}M\} \tag{2}$$

That is L_k is the number of shocks until the first deadly shock occurred following the $(k-1)^{st}$ failure. Then

$$X_k = \sum_{i=1}^{L_k} W_{ki} \tag{3}$$

And L_k follows a geometric distribution $G(P_k)$, with

$$P(L_k = k) = P_k(1-P_k)^{k-1}, k = 1,2 \dots \dots \tag{4}$$

Where

$$P_k = P(D_{ki} > a^{k-1}M) \tag{5}$$

By equation (4) , we have

$$E(L_k) = \frac{1}{P_k} \tag{6}$$

As $\{W_{ki}, i = 1,2,3 \dots \dots\}$ and $\{D_{ki}, i = 1,2,3 \dots .\}$ are independent, obviously L_k and $\{W_{ki}, i = 1,2,3 \dots \dots\}$ are independent.

Then,at that point, from conditions (3), (4) and by Wald's equation. We have

$$\lambda_k = E(X_k) = E(L_k)E(W_{ki}) = \frac{\lambda}{P_k} \tag{7}$$

Since $a \leq 1$ from equations(5) and(7). We can derive that λ_k is decreasing in k.

From equation (1),

$$C(N) = \frac{E\left(c\sum_{k=1}^{N-1} Y_k + R - r\sum_{k=1}^{N} X_k\right)}{E\left(\sum_{k=1}^{N} X_k + \sum_{k=1}^{N-1} Y_k + Z\ \right)}$$

$$= \frac{c\left(\sum_{k=1}^{N-1} E(Y_k)\right) + R - r\sum_{k=1}^{N} \lambda_k\Big)}{\sum_{k=1}^{N} \lambda_k + \left(\sum_{k=1}^{N-1} Y_k\right) + \tau}$$

$$C(N)=\frac{c\left(\sum_{k=1}^{N-1}\frac{\mu}{k^{\alpha}}\right)+R-r\sum_{k=1}^{N}\lambda_k}{\sum_{k=1}^{N}\lambda_k+\left(\sum_{k=1}^{N-1}\frac{\mu}{k^{\alpha}}\right)+\tau} \tag{8}$$

4 The optimal substitution strategy N^*

In this part, we decide an optimal substitution strategy for limiting $C(N)$

From condition (8), we have

$$C(N)=\frac{(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+R+r\tau}{h(N)}-r \tag{9}$$

Where $$h(N)=\sum_{k=1}^{N}\lambda_k+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau$$

To accuire the optimal strategyN^*, we want to explore the distinction between

$C(N+1) and\ C(N)$.

$$C(N+1)-C(N)=\left[\frac{(c+r)\mu\left(\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+R+r\tau}{h(N+1)}-r\right]-\left[\frac{(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+R+r\tau}{h(N)}-r\right]$$

$$=\frac{\begin{bmatrix}\left[\sum_{k=1}^{N}\lambda_k+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right]\left[(c+r)\mu\left(\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+R+r\tau\right]-\\ \left[\sum_{k=1}^{N+1}\lambda_k+\mu\left(\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+\tau\right]\left[(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+R+r\tau\right]\end{bmatrix}}{h(N+1)h(N)}$$

$$=\frac{\begin{bmatrix}(c+r)\mu\sum_{k=1}^{N}\lambda_k\left(\frac{1}{N^{\alpha}}\right)-\lambda_{N+1}(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)-R\lambda_{N+1}\\ -r\tau\lambda_{N+1}+\tau(c+r)\mu\left(\frac{1}{N^{\alpha}}\right)-R\mu\left(\frac{1}{N^{\alpha}}\right)-\mu r\tau\left(\frac{1}{N^{\alpha}}\right)\end{bmatrix}}{h(N+1)h(N)}$$

$$=\frac{\left[(c+r)\mu\left(\sum_{k=1}^{N}\lambda_k-N^{\alpha}\lambda_{N+1}\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right)-\lambda_{N+1}(R+r\tau)-\mu\left(\frac{1}{N^{\alpha}}\right)(R+r\tau)\right]}{h(N)h(N+1)}$$

$$C(N+1)-C(N)=\frac{\left[(c+r)\mu\left(\sum_{k=1}^{N}\lambda_k-N^{\alpha}\lambda_{N+1}\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right)-(R+r\tau)(N^{\alpha}\lambda_{N+1}+\mu)\right]}{h(N)h(N+1)} \tag{10}$$

As the denominator of $C(N+1)-C(N)$ is generally certain. Obviously the indication of $C(N+1)-C(N)$ is equivalent to the indication of its numerator.

Subsequently we present the auxillary function $B(N)$ as follows.

$$B(N)=\frac{(c+r)\mu\left(\sum_{k=1}^{N}\lambda_k-N^{\alpha}\lambda_{N+1}\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right)}{(R+r\tau)(N^{\alpha}\lambda_{N+1}+\mu)} \tag{11}$$

Subsequently we have the accompanying lemma.

Lemma 4.1. $C(N+1)>C(N)\Leftrightarrow B(N)>1$

$$C(N+1) = C(N) \Leftrightarrow B(N) = 1$$

$$C(N+1) < C(N) \Leftrightarrow B(N) < 1 \tag{12}$$

Lemma (4.1) shows that the monotonicity of $C(N)$ can be determined by the value of $B(N)$

From equation (11) we have

$$B(N+1) - B(N) = \begin{bmatrix} \left(\frac{(c+r)\mu\left(\sum_{k=1}^{N+1} \lambda_k - (N+1)^\alpha \lambda_{N+2}\left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + \tau\right)}{(R+r\tau)((N+1)^\alpha \lambda_{N+2} + \mu)} \right) - \\ \left(\frac{(c+r)\mu\left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1}\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right)}{(R+r\tau)(N^\alpha \lambda_{N+1} + \mu)} \right) \end{bmatrix}$$

Let

$$A(N) = \frac{(c+r)\mu}{(R+r\tau)((N+1)^\alpha \lambda_{N+2} + \mu)(N^\alpha \lambda_{N+1} + \mu)}$$

Now,

$$B(N+1) - B(N) = A(N) \begin{bmatrix} (N^\alpha \lambda_{N+1} + \mu)\left(\sum_{k=1}^{N+1} \lambda_k - (N+1)^\alpha \lambda_{N+2}\left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + \tau\right) \\ -((N+1)^\alpha \lambda_{N+2} + \mu)\left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1}\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right) \end{bmatrix}$$

$$= A(N) \begin{bmatrix} \mu\lambda_{N+1} + \lambda_{N+1} N^\alpha \lambda_{N+1} - \lambda_{N+1}(N+1)^\alpha \lambda_{N+2} - \mu(N+1)^\alpha \lambda_{N+2}\left(\frac{1}{N^\alpha}\right) \\ +(N^\alpha \lambda_{N+1} - \lambda_{N+2}(N+1)^\alpha)\left(\mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \sum_{k=1}^{N} \lambda_k + \tau\right) \end{bmatrix}$$

$$= A(N) \begin{bmatrix} \left(\mu\left(\frac{1}{N^\alpha}\right) + \lambda_{N+1}\right)(\lambda_{N+1} N^\alpha - \lambda_{N+2}(N+1)^\alpha) + \\ (\lambda_{N+1} N^\alpha - \lambda_{N+2}(N+1)^\alpha)\left(\mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \sum_{k=1}^{N} \lambda_k + \tau\right) \end{bmatrix}$$

$$= A(N)\left[(\lambda_{N+1} N^\alpha - \lambda_{N+2}(N+1)^\alpha)\left(\mu\left(\frac{1}{N^\alpha}\right) + \lambda_{N+1} + \mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \sum_{k=1}^{N} \lambda_k + \tau\right)\right]$$

$$B(N+1) - B(N)$$
$$= \frac{(c+r)\mu\left[(\lambda_{N+1} N^\alpha - \lambda_{N+2}(N+1)^\alpha)\left(\mu\left(\frac{1}{N^\alpha}\right) + \lambda_{N+1} + \mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \sum_{k=1}^{N} \lambda_k + \tau\right)\right]}{(R+r\tau)((N+1)^\alpha \lambda_{N+2} + \mu)(N^\alpha \lambda_{N+1} + \mu)}$$

Because λ_k is decreasing in k and $\alpha < 0$,this demonstrates that $B(N)$ is not decreasing in$N$.

We can derive the following theorem using lemma (4.1).

Theorem 4.1. The optimal substitution strategy not entirely settled by

$$N^* = \min\{N \,/\, B(N) \geq 1\} \tag{13}$$

Besides, the optimal replacement strategy N^* is special if and provided that $B(N^*) > 1$.

5 A Computational example

We concentrate on a mathematical model with the supposition that D_{11} has an exponential distribution with expectation β.

Then $$F(x) = P(D_{11} \leq x) = 1 - e^{-\left(\frac{1}{\beta}\right)x}$$

From equation (5), we have

$$P_k = P(D_{ki} > a^{k-1}M) = e^{-\left(\frac{1}{\beta}\right)a^{k-1}M} \tag{14}$$

Then by equation (7), we have

$$\lambda_k = E(X_k) = \frac{\lambda}{P_k} = \lambda e^{\left(\frac{1}{\beta}\right)a^{k-1}M} \tag{15}$$

Substitute equation (15) in equation (9) and (11), the exact expression for $C(N)$ and $B(N)$ are

$$C(N) = \frac{\left[(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+R+r\tau\right]}{\left[\lambda\sum_{k=1}^{N} e^{\left(\frac{1}{\beta}\right)a^{k-1}M}+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right]} - r \tag{16}$$

$$B(N) = \frac{(c+r)\mu\left[\lambda\sum_{k=1}^{N} e^{\left(\frac{1}{\beta}\right)a^{k-1}M}+\tau-N^{\alpha}\lambda e^{\left(\frac{1}{\beta}\right)a^{N}M}\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right]}{(R+r\tau)\left[\mu+N^{\alpha}\lambda e^{\left(\frac{1}{\beta}\right)a^{N}M}\right]} \tag{17}$$

Let

$$c = 6, \mu = 10, r = 10, \tau = 10, a = 0.95, R = 6000, \lambda = 10, \beta = 10, \alpha = -0.98, M = 20$$

The numerical results are presented in Table 1 and the corresponding figures are plotted in Figure 1 and Figure 2 respectively.

N	$C(N)$	$B(N)$	N	$C(N)$	$B(N)$
1	62.71378243	0.02862918	10	3.64595261	0.69378903
2	28.94257705	0.07507138	11	3.43664941	0.77152329
3	17.25365310	0.13569171	12	3.31351515	0.84659007
4	11.59934997	0.20654302	13	3.25081222	0.91883968
5	8.42880843	0.28411102	14	**3.23075356**	**1.00012229**
6	6.50549464	0.36552940	15	3.24082219	1.05475939
7	5.28622768	0.44859551	16	3.27208457	1.11851447
8	4.49555734	0.53169253	17	3.31809507	1.17958295
9	3.97953895	0.61368059	18	3.37416518	1.23807733

Table : The values of $C(N)$ and $B(N)$ for different values of N.

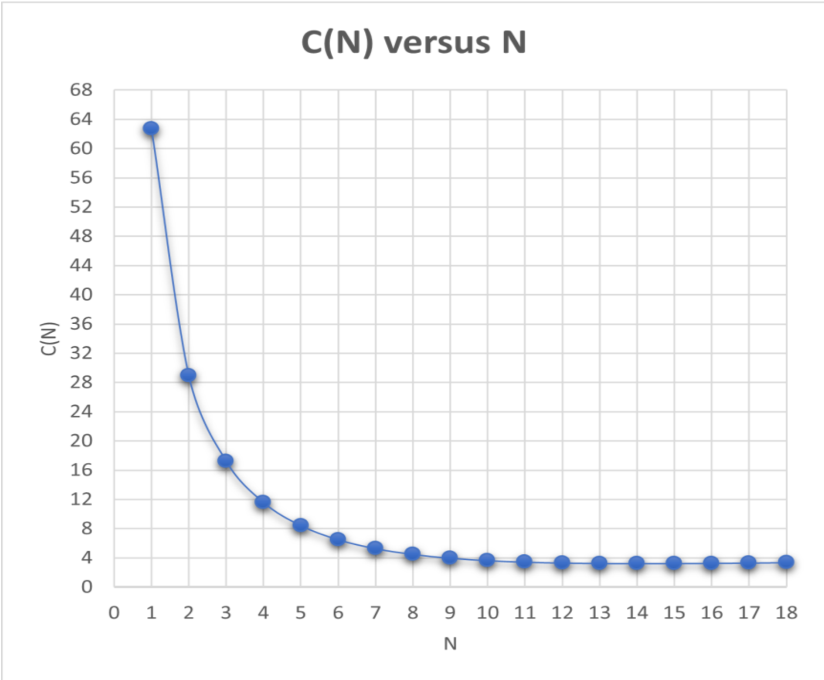

Figure 1

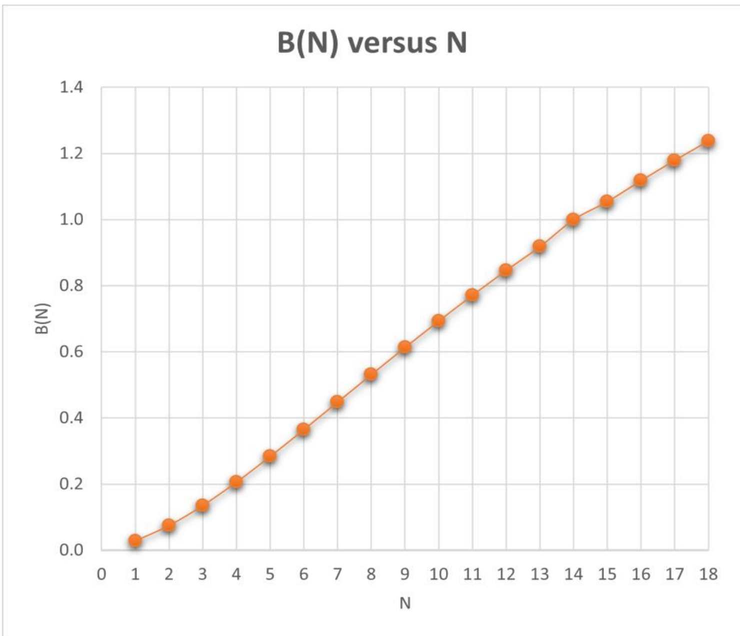

Figure 2

Clearly $C(14) = 3.23075356$ is the minimum of the long run average cost. On the other hand $B(14) = 1.00012229 > 1$ and $min\{N/B(N) \geq 1\} = 14$

6 Conclusion

By considering an EES maintenance model for a systems disintegrating, an exact expression for the long-run mean cost for each unit time under a monotone process is derived. Perceptionally, an optimal substitution strategy N^* for reducing the long-run mean cost for each unit time is determined. A computational example is given to illustrate the methodology developed in this research work. As a system parameter, the threshold value can be approximated. The project's goal is to carry on. As a result, for a system that has been upgraded, we can develop an optimal substitution strategy N.

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