Modeling the Behavior of Tourist Agencies in the Market of Providing Services

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***Abstract* – The features of mathematical models describing the behavior strategies of tourist agencies in the market of services. It was found that in the market of tourist services of any entity it must act in accordance with the established rules of behavior and interaction of participants in this market. The mathematical models of behavior strategies and the interaction of two travel agencies in the provision of one service have been developed. The theoretical results obtained allowed to determine the reaction of the behavior of one firm on the actions of its competitors, to establish a strategy of behavior and Shtakelberg equilibrium situations, Cournot and Nash equilibrium points. The results of modeling strategies for imperfect and perfect competition of travel companies in the market for the provision of services made it possible to make sound management decisions.**

Keywords – market of tourist services; agencies behaviour and agencies interaction; perfect and imperfect competition; agencie response to actions of a competitor; game theory.

I. INTRODUCTION

Today agencies in the business of providing services must act in accordance with established rules of behavior and interact with participants in their sector [1]. These rules depend on factors such as the number of participants, the presence of impediments to enter and exit the sector, the degree of influence of each subject on the whole market in general with special focus on one's business activities in particular. Rivalry among existing travel agencies often comes down to agencies resorting to every available means and methods to achieve a better position using tactics like pricing policy, providing services, promoting its services and outdo the competition through extensive advertising. The basis of the market relations is competition between tourist agencies that provide their original services, competition for customers with the view to getting the maximize financial results – income, profit [7].

Perfect competition represents a market, that has a large number of tourist agencies that provide approximately similar services at almost the same price. At the same time, imperfect competition has several variants in which competition between tourist agencies is limited by various factors: in a monopoly there is only one big agency that provides its services at a relatively high price, while the entry of other agencies and their exit from it is almost impossible; in an oligopoly, there are a few relatively large agencies which often are part of a conspiracy. They establish certain obstacles hindering the entrance of competitors into the marketplace causing problems, etc. It is believed [5], that perfect competition (not the ideal, of course) dominates most markets providing tourist services. It is most conducive for the government, that seeks to ensure market principles for generating business and engages in less interference in the activities of tourist agencies, as required in imperfect competition, especially in the case of a monopoly [4].

But the current process of transforming the economic system in general and in the tourism industry in particular taking place in Ukraine at its present stage of development, accompanied by manifestation the number of problems in choosing the optimal strategy of behavior and interaction of agencies in the market providing services. In particular, available literature for modeling admissible behavior strategies of competitive agencies in the market providing tourist services is practically nil: the mathematical models that describe the behavior strategies of two or more competitive agencies while providing one and different services; no corresponding models describing strategies interaction between two tourist agencies providing one service; not developed the model strategies imperfect and perfect competition for tourist agencies providing services. All this calls for a need to study forces market focusing on possible competition and behavior of tourist agencies while providing appropriate services and making informed decisions [2, 3].

II. MATHEMATICAL MODEL OF PROVIDING ONE TOURISM SERVICE

We consider the features of constructing the mathematical model of behavior strategies of competitive agencies providing tourist services by the mechanism of resolve the conflict situations between participants with opposing interests, the mathematical model of which is a game with non-zero sum. Initially considered a simple case of oligopoly – duopoly, that is, when the market providing tourist services involves only two competing agencies [10, p. 311].

Let's consider on the market providing services there are two tourist agencies that offer vacationers the same travel service. If x_1 i x_2 – the volume of tourism services, provided under the first and second agencies, then the market value of tourism services (v), obviously, will depend on their total proposals, namely $v = v(x_1 + x_2)$. Assume, that this dependence is linear $v = v(x_1 + x_2) = a - b(x_1 + x_2)$, where a, b – accordingly are constant and variable components of the volume providing tourist services: $a > 0$, $b > 0$. Also assume that the expenditure of tourist agencies are described by the same linear functions which depend on the volume providing tourist service $W(\tilde{X}) = \{w(x_i) = cx_i + d,\, i = \overline{1,2}\}$, where: $w(x_i)$ – total expenditures, that uses by the i-th agency during providing x_i units of tourism services; c, $d > 0$ – accordingly variable and fixed expenditures. Our assumptions about the same functions that describe expenditures of each tourism agency means that both competing agencies use the same technological processes to provide tourism services. Obviously, the profit of i-th tourist agencies (p_i) will depend on the volume of providing tourist services by two competing agencies:

$$p_i(x_1, x_2) = v(x_1 + x_2) \cdot x_i - w(x_i) = (a - c - b(x_1 + x_2)) \cdot x_i - d,\ i = \overline{1,2}.$$

We introduce a notation $x_0 = (a - c)/b$ and rewrite this formula in this form:

$$\tilde{P}(x_1, x_2) = \left\{ p_i(x_1, x_2) = b \cdot (x_0 - (x_1 + x_2)) \cdot x_i - d,\ i = \overline{1,2} \right\}. \quad (1)$$

From this formula we see that x_0 – it is a total volume of tourist services provided by agencies in which the profits of each agency is negative and equal to regular charges, taken with the opposite sign. This means, that in case, when a proposal of total volume providing tourist services will be x_0, then the profit of each agency providing services cover only variable costs. Obviously, each of the competing agencies seek to choose their volume of providing tourist services (x_i) so, as to obtain the maximum profit. Let's try to explore how 1-st tourist agencies will respond to a known volume of providing services by a second agency. According to the formula (1), the profit of the first tourist agencies is

$$p_1(x_1, x_2) = b \cdot (x_0 - (x_1 + x_2)) \cdot x_1 - d\ . \quad (2)$$

In general, the 1-st tourist agencies believes that the volume of providing services x_2 competitive agency depends on its own volume of providing services x_1. Substituting $x_1 = x_2(x_1)$ in formula (2), we obtain the following expression $p_1(x_1) = b \cdot (x_0 - (x_1 + x_2(x_1))) \cdot x_1 - d$. Now let's find x_1 from the condition finding maximum profit of the first tourist agencies [6], that is the maximum value of the function $p_1(x_1)$:

$$\begin{cases} \dfrac{dp_1(x_1)}{dx_1} = 0; \\ \dfrac{d^2 p_1(x_1)}{dx_1^2} < 0, \end{cases} \Leftrightarrow \begin{cases} b \cdot \left(x_0 - x_2(x_1) - \left(2 + \dfrac{dx_2(x_1)}{dx_1} \right) \cdot x_1 \right) = 0; \\ -b \cdot \left(2 + 2\dfrac{dx_2(x_1)}{dx_1} + x_1 \dfrac{d^2 x_2(x_1)}{dx_1^2} \right) < 0. \end{cases}$$

The above formula (3), which determines the *reaction of the 1-st tourist agencies* on certain volume of providing services by the 2-nd agency (not necessarily constant). Similarly, we define the reaction of the 2-nd tourist agencies to the actions of the 1-st agency, so that we get formula (4).

$$x_1^*(x_2) = \frac{x_0 - x_2(x_1)}{2 + \dfrac{dx_2(x_1)}{dx_1}},\ x_2^*(x_1) = \frac{x_0 - x_1(x_2)}{2 + \dfrac{dx_1(x_2)}{dx_2}}. \quad (3), (4)$$

In formulas (3)–(4) dx_i/dx_j – it is predictable change of volume providing tourist services by i-th agency, which is associated with an increase of unit volume of providing services by a competing agency. As a result of implementing the developed mathematical model of behavior strategies of two competing agencies providing the one tourism service allows comparing the reaction of one agency to the actions of its competitor.

III. Model of strategies for the interaction of two travel companies

Let each tourist agencies knows exactly the volume of providing service of its competitor and consider this volume unchanged over a production cycle [8]. This means, that in formulas (3)–(4) we obtain $dx_1(x_2)/dx_2 = dx_2(x_1)/dx_1 = 0$, so the function of reaction of the 1-st tourist agencies to known *constant volume of providing service by the 2-nd agency* with be taking into account (3) will be determined by (5). Similarly, when taking into account (4) we determine reaction of the 2-nd tourist agencies to the actions of the 1-st agency on the condition under which the 2-nd agency believes that the volume of providing service by the first agency is constant (see (6)).

$$x_1^*(x_2) = \frac{x_0 - x_2(x_1)}{2},\ x_2^*(x_1) = \frac{x_0 - x_1(x_2)}{2}. \quad (5), (6)$$

We assume that the *production cycles* of both tourist agencies match each other, so let's consider a few of such consecutive production cycles. In the first production cycle the volume of providing services by tourist agencies will be the same and will be in accordance with $x_1(1)$ i $x_2(1)$. At the same time, agencies in each of the next production cycle have set their volume of providing services by formulas (5) and (6), assuming that in accordance the volume of providing service by each competitive agency will be the same as in the previous cycle. The total volume of providing services by the two tourist agencies in equilibrium point of Cournot will be: $x_1^C + x_2^C = x_0/3 + x_0/3 = 2x_0/3$, and the cost of each tourist services at this point amounts to:

$$v^C = v(x_1^C + x_2^C) = a - b \cdot (x_1^C + x_2^C) = a - b \cdot \frac{2x_0}{3}.$$

Profit of the 1-st tourist agencies amounts to

$$p_1^C = p_1(x_1^C, x_2^C) = b \cdot \left(x_0 - (x_1^C + x_2^C) \right) \cdot x_1^C - d = b \cdot \frac{x_0^2}{9} - d,$$

and profit of the 2-nd tourist agencies is similar amounts to: $p_2^C = p_2(x_1^C, x_2^C) = b \cdot x_0^2/9 - d$. Let's say that the 1-st tourist agencies *intentionally tells* its competitor that their volume of providing service x_1, a competitive firm, knowing it, will calculate their volume of providing service according to the formula (6) – that considers the volume of providing service by 1-st tourist agencies is constant. Then the profit of 1-st tourist agencies amounts to:

$$p_1(x_1, x_2^*(x_1)) = b \cdot \left(\frac{x_0 - x_1}{2} \right) \cdot x_1 - d\ .$$

But before notifying their competition of their volume of providing services, the 1-st tourist agencies can choose a value x_1, that its profit $p_1(x_1, x_2^*(x_1))$ will be the largest, that is $p_1(x_1, x_2^*(x_1)) \to \max$. Terms of finding the maximum value of this function have the following form:

$$\begin{cases} \dfrac{dp_1}{dx_1} = 0; \dfrac{d^2 p_1}{dx_1^2} < 0, \end{cases} \Leftrightarrow \begin{cases} b \cdot \left(\dfrac{x_0}{2} - x_1 \right) = 0; \\ -b < 0, \end{cases} \Leftrightarrow x_1^{\mathrm{rS}} = \frac{x_0}{2}. \qquad (7)$$

This behavior strategy adopted by the 1-st tourist agencies is called *Stackelberg strategy* [8]. If the 2-nd tourist agencies was to act like the 1-st agency, then according to formula (6), it will elect to adopt the following formula:

$$x_2^{\mathrm{rS}} = \frac{x_0 - x_1^{\mathrm{rS}}}{2} = \frac{x_0}{4}, \qquad (8)$$

that will provide to it getting maximum profit, under the condition, that the proposal of the competing agency is $x_1^{\mathrm{pШ}}$, then such a situation is called *Stackelberg equilibrium*. In Stackelberg point of equilibrium the total volume of providing services by two tourist agencies amounts to

$$x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}} = \frac{x_0}{2} + \frac{x_0}{4} = \frac{3x_0}{4} > x_1^{\mathrm{C}} + x_2^{\mathrm{C}},$$

and their cost $v^{\mathrm{rS}} = v(x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}}) = a - b \cdot (x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}}) = a - b \cdot \dfrac{3x_0}{4} < v^{\mathrm{C}}$.

Thus, the 1-st tourist agencies will get profits

$$p_1^{\mathrm{rS}} = p_1(x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}}) = b\left(x_0 - (x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}})\right)x_1^{\mathrm{rS}} - d = b\frac{x_0^2}{8} - d > p_1^{\mathrm{C}},$$

and profit of the 2-nd tourist agencies in Stackelberg point of equilibrium will be:

$$p_2^{\mathrm{rS}} = p_2(x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}}) = b\left(x_0 - (x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}})\right)x_2^{\mathrm{rS}} - d = b\frac{x_0^2}{16} - d < p_2^{\mathrm{C}}.$$

Consequently, in Stackelberg point of equilibrium the profit of the 2-nd tourist agencies is significantly smaller than in Cournot point of equilibrium, so the 2-nd agency may not want to "go on a leash" as the 1-st agency and receive such a so small profit. Obviously, the 2-nd agency can decide on its volume of providing tourist services according to Stackelberg strategy. In a situation, where *both agencies in the business of providing the same tourist services* are acting in accordance with the Stackelberg strategy, called *Stackelberg disequilibrium* [8].

If the 1-st tourist agencies believes that the competing agency, knowing the volume of providing service by the 1-st firm, it will choose its volume of providing service by formula (6), ie $x_2(x_1) = (x_0 - x_1(x_2))/2$, so the predictable change will be $dx_2(x_1)/dx_1 = -1/2$, in resulting of it, formula (3) takes the following form:

$$x_1^*(x_2) = \frac{x_0 - x_2(x_1)}{2 - 1/2} = \frac{x_0 - x_2(x_1)}{3/2} = \frac{2}{3}(x_0 - x_2(x_1)). \qquad (9)$$

Typically, solution of linear equations systems, consisting of expressions (6) and (9) is the Stackelberg equilibrium point, which is determined by (7)–(8), namely:

$$x_1^*(x_2) = \frac{2}{3}(x_0 - x_2(x_1)); \; x_2^*(x_1) = \frac{x_0 - x_1(x_2)}{2}.$$

If necessary, an interested reader can be convinced of it yourself. In *Stackelberg disequilibrium* the 2-nd tourist agencies chooses its volume of providing service not by formula (6), by the following formula

$$x_2^*(x_1) = \frac{2}{3}(x_0 - x_1(x_2)), \qquad (10)$$

which is similar to the formula (9). Thus the Stackelberg disequilibrium point is determined from system of linear equations (9) and (10), the solution of which will be:

$$\begin{cases} x_1^*(x_2) = \dfrac{2}{3}(x_0 - x_2) \\ x_2^*(x_1) = \dfrac{2}{3}(x_0 - x_1) \end{cases} \Leftrightarrow \begin{cases} 3x_1 = 2x_0 - 2x_2; \\ 3x_2 = 2x_0 - 2x_1, \end{cases} \Leftrightarrow \begin{cases} x_1^{\mathrm{nS}} = \dfrac{2x_0}{5}; \\ x_2^{\mathrm{nS}} = \dfrac{2x_0}{5}. \end{cases}$$

The total volume of providing services by two tourist agencies in Stackelberg disequilibrium point amounts to:

$$x_1^{\mathrm{nS}} + x_2^{\mathrm{nS}} = \frac{2x_0}{5} + \frac{2x_0}{5} = \frac{4x_0}{5} > x_1^{\mathrm{rS}} + x_2^{\mathrm{rS}},$$

and their cost $v^{\mathrm{nS}} = v(x_1^{\mathrm{nS}} + x_2^{\mathrm{nS}}) = a - b(x_1^{\mathrm{nS}} + x_2^{\mathrm{nS}}) = a - b\dfrac{4x_0}{5} < v^{\mathrm{rS}}$, while the profit of each of the tourist agencies amounts to:

$$p_1^{\mathrm{nS}} = p_1(x_1^{\mathrm{nS}}, x_2^{\mathrm{nS}}) = b \cdot \left(x_0 - (x_1^{\mathrm{nS}} + x_2^{\mathrm{nS}})\right)x_1^{\mathrm{nS}} - d = b \cdot \frac{2x_0^2}{25} - d;$$
$$p_2^{\mathrm{nS}} = p_2(x_1^{\mathrm{nS}}, x_2^{\mathrm{nS}}) = b \cdot \left(x_0 - (x_1^{\mathrm{nS}} + x_2^{\mathrm{nS}})\right)x_2^{\mathrm{nS}} - d = b \cdot \frac{2x_0^2}{25} - d.$$

If two competing agencies merge into a single agency, such an association creates a *monopoly*. If x – is volume of providing tourist services by one monopoly firm, so its cost will be $p(x) = a - bx$ and, as expenditure is described by function $w(x) = cx + d$, the profits of one monopoly agency calculated as $p(x) = v(x) \cdot x - w(x) = b \cdot (x_0 - x)x - d$. The optimal volume of providing tourist services by one monopoly agency is determined by finding the maximum profit:

$$\begin{cases} \dfrac{dp}{dx} = 0; \dfrac{d^2 p}{dx^2} < 0, \end{cases} \Leftrightarrow \begin{cases} b \cdot (x_0 - 2x) = 0; \\ -2b < 0, \end{cases} \Leftrightarrow x_{\mathrm{m}} = \frac{x_0}{2}.$$

With this proposal of total volume of providing tourist services (which is less than total proposals in Cournot equilibrium point) its cost will be $v(x_{\mathrm{m}}) = a - bx_0/2 > v^{\mathrm{C}}$, and the profit of one monopoly agency will be

$$p(x_{\mathrm{m}}) = b \cdot (x_0 - x_{\mathrm{m}})x_{\mathrm{m}} - d = b \cdot \left(x_0 - \frac{x_0}{2}\right)\frac{x_0}{2} - d = b \cdot \frac{x_0^2}{4} - d.$$

Antimonopoly laws can prohibit the formation of monopolies in cases where it is not profitable for ordinary consumers of tourism services. In such cases, tourist agencies can form a *cartel*, that is to join the conspiracy, agreeing to their volume of providing services in order to maximize profit. In this case, both tourist agencies can negotiate to maximize their joint profits $p_{1+2}(x_1, x_2) = b \cdot (x_0 - (x_1 + x_2)) \cdot (x_1 + x_2) - 2d,$ and then divide it among themselves in certain proportions. Thus, the condition of finding the maximum joint profit by two tourist agencies will be:

$$\begin{cases} \frac{\partial p_{1+2}}{\partial x_1} = 0; \frac{\partial p_{1+2}}{\partial x_2} < 0, \end{cases} \Leftrightarrow b \cdot (x_0 - 2(x_1 + x_2)) = 0 \Leftrightarrow x_1 + x_2 = \frac{x_0}{2} .$$

So, maximum joint profit from providing one service, which tourist cartel plans to get, is achieved at any point in the segment line, is defined in the equation $x_1 + x_2 = x_0 / 2$ when $x_1 \geq 0$, $x_2 \geq 0$. Consider now the strategy *of maximizing joint profits* from providing one tourist service, based on the cartel model [6, 10]. Isoprofit *i*-th tourist agencies called line, on which profits of this agency is constant (ie *line of profit level i*-th tourist agencies). The equation of isoprofit for the 1-st tourist agencies takes the following form: $p_1(x_1, x_2) = \pi_1^0 = const$ or $b \cdot (x_0 - (x_1 + x_2)) \cdot x_1 - d = \pi_1^0$. First, consider the situation where $\pi_1^0 = -d$. With $p_1(x_1, x_2) = -d \Leftrightarrow b \cdot (x_0 - (x_1 + x_2)) \cdot x_1 = 0$, from where $x_1 + x_2 = x_0$, ie the volume of providing service by the 1-st tourist agencies amounts to $x_1 = 0$. Thus, the value of joint profit $\pi_1^0 = -d$ corresponds such isoprofit

$$\{(x_1, x_2) | (x_1 = 0, x_2 \in [0, x_0]) \cup (x_1 \in [0, x_0], x_2 = x_0 - x_1]) \} .$$

In the case, when joint profit $\pi_1^0 > -d$, the volume of providing services by the 2-nd tourist agencies will be:

$$x_2 = x_0 - x_1 - \frac{\pi_1^0 + d}{b x_1} . \qquad (11)$$

Similarly, we can derive an equation of isoprofit for the 2-nd tourist agencies $p_2(x_1, x_2) = b \cdot (x_0 - (x_1 + x_2)) \cdot x_2 - d = \pi_2^0 = const$, and also determine from this equation (with $\pi_2^0 > -d$) the volume of providing services by the 1-st tourist agencies, which will be:

$$x_1 = x_0 - x_2 - \frac{\pi_2^0 + d}{b x_2} . \qquad (12)$$

The points at which none of the tourist agencies can achieve an increase in its profits without reducing profits of competitive agencies are *optimal by Pareto* [6]. From a geometrical point of view the set of this points are forming *contractual curve*, formed by touching points of two isoprofit for two tourist agencies. The condition of touching two isoprofit (i.e., *level lines for getting joint profit*) is equivalent to collinearity of two gradients: $grad\ p_1 \| grad\ p_2$ or $\frac{\partial p_1}{\partial x_1} \cdot \frac{\partial x_2}{\partial p_1} = \frac{\partial p_2}{\partial x_1} \cdot \frac{\partial x_2}{\partial p_2}$.

Substituting here formulas of partial derivatives, will have:

$$\frac{\partial p_1}{\partial x_1} = b(x_0 - 2x_1 - x_2), \frac{\partial p_1}{\partial x_2} = -b x_1,$$
$$\frac{\partial p_2}{\partial x_1} = -b x_2, \frac{\partial p_2}{\partial x_2} = b(x_0 - x_1 - 2x_2).$$

As a result we obtain

$$\frac{b \cdot (x_0 - 2x_1 - x_2)}{-b x_1} = \frac{-b x_2}{b \cdot (x_0 - x_1 - 2x_2)} \Leftrightarrow$$
$$\Leftrightarrow (x_0 - 2(x_1 + x_2))(x_0 - (x_1 + x_2)) = 0.$$

Since the total volume of providing tourist services $x_1 + x_2$ is always less than x_0 (otherwise both tourist agencies receive negative profit (–*d*)), the last formula shows that the contract curve is determined by the condition $x_0 - 2(x_1 + x_2) = 0$ or $x_1 + x_2 = x_0 / 2$. Earlier, the same condition defined maximum joint profits of agencies in the tourism cartels, i.e. maximum joint profits achieved on the contract curve. This curve is the set of points, at which one chooses for interaction, tourist agencies can decide only in the process of negotiation.

So, the developed models of strategies the interaction between two tourist agencies in the market providing one service, that enabled it to define the strategy of behavior and situation (not) equilibrium of Stackelberg, point of equilibrium of Cournot and investigate the stability of equilibrium states.

IV. Models of strategies the imperfect and perfect competition of tourist agencies in the business of providing services

Assumptions models equilibrium of Cournot that tourist agencies make decisions regarding the amount of services they will provide, considering that some changes of their amount of providing service will not influence the amount of providing service of competitive agencies, in a case of a duopoly it is rather naive. Conversely, in the case of *competition*, when there are a lot of participants providing services it is indeed possible to believe that the actions of one of the tourist agencies will not influence of the actions of others [9].

The competitiveness of agencies providing tourist services is determined by the framework [11], within which some agencies are able to influence the market, i.e. on terms of providing its service, primarily on its cost. The less individual agencies are influence by the market, where they provide their services, the more competitive market is considered. The highest level of competitive agencies in the business of providing tourist services is achieved when an individual agency does not influence in it at all. This is possible only when many agencies are providing services so that each of them in particular does not influence the value of the services, i.e. accepts it is determined by supply and demand. This is called the fully competitive market, and tourist agencies, operating in its terms, does not compete with each other. If individual tourist agencies have the ability to influence conditions of realization of their services (primarily on their value), they compete with each other, but markets where this opportunity is realized, is not considered completely competitive [12].

Let's write *the Cournot generalization equilibrium* in a case of N market participants providing tourist services [7]:

$$x_i^{\mathrm{C}} = \frac{x_0}{N+1},\ i = \overline{1,N};\ v^{\mathrm{C}} = a - b\frac{N}{N+1}x_0;\ \sum_{i=1}^{N} x_i^{\mathrm{C}} = \frac{N}{N+1}x_0;$$
$$p_i(x_j^{\mathrm{C}},\ j = \overline{1,N}) = \frac{bx_0^2}{(N+1)^2} - d,\ i = \overline{1,N}. \quad (13)$$

We hope that interested readers can independently deduce these formulas. In the case of *perfect competition*, i.e. when $N \to \infty$, then the limit transition in expressions (13)–(16) we get that individual volumes providing services competitive agencies $x_i^{\mathrm{C}} \to 0,\ i = \overline{1,N}$, and cost provided services amounts to $v^{\mathrm{C}} \to a - bx_0 = c$, that is equal to variable expenditures (because $x_0 = (a-c)/b$), while the total volume providing tourism services by all competitive agencies amounts to $\sum_{i \in N} x_i^{\mathrm{C}} \to x_0$, and the profit of each competitive agency amounts to

$$p_i(x_j^{\mathrm{C}},\ j = \overline{1,N}) \to -d,\ i = \overline{1,N}\ . \quad (14)$$

This means, that every tourist agencies in this case provides so small a volume of service that this service does not influence its overall value; equilibrium cost of providing tourist services, thus, equals marginal expenditures. Therefore, developed models of strategies the imperfect and perfect competition of travel agencies in the business of providing services are enabled to make informed management decisions.

V. Conclusions

1. Considered the features of constructing mathematical models that describe the behavior of the different strategies of competitive agencies in the business of tourist services. These rules depend on factors such as the number of market participants, the presence of impediments to entry and exit from it, the degree of influence of each subject on the whole market in general and at its own segment of business activities in particular.

2. Developed mathematical model of behavior strategies of two competing agencies providing one tourism service, the results of which allows it to define the reaction of one agency in relation to the actions of its competitor. The received mathematical formulas by which we can determine the reaction of one tourist agencies for a certain amount of providing service to another agency, and vice versa.

3. We have developed a models of strategies highlighting the interaction between two tourist agencies on the market providing one service, that enabled us to define the strategy of behavior and situation (not) equilibrium of Stackelberg, point of equilibrium of Cournot and investigate the stability of equilibrium states.

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