Local Adaptive Wavelet Threshold Denoising Based on Elliptic Directional Window and Edge Detection

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Abstract—**Due to the sampling method for the wavelet coefficients of image can better adapting to its main directional characteristics, and the edge detection protects the edge information of the image, a local adaptive wavelet denoising method based on elliptic direction window and edge detection is proposed in this paper. The method first performs wavelet decomposition for image, and performs edge detection on the wavelet coefficients. Then, the wavelet coefficients of image are sampled by the elliptic directional window, and the local threshold of it is calculated. Next, the wavelet coefficients are quantized by soft threshold function. Finally, the denoised image is obtained by inverse wavelet transformation. In addition to be noted that a weight less than 1 is multiplied to reduce the threshold amplitude as much as possible to preserve the edge features of the image. In order to validate the performance of the proposed denoising method, four standard gray-scale test images are employed and the denoising results are compared with the Local Wiener Filtering with Directional Windows (LWFDW). The experimental results show that the method proposed in this paper performs better in terms of numerical indicators, is smoother in visual and has fewer pseudo-Gibbs phenomena than the LWFDW.**

Keywords-component; Wavelet transform; elliptic directional window; edge detection; denosing; local threshold

I. INTRODUCTION

Since different types of noise are inevitably introduced in the processes of image formation and transmission, image denoising is a necessary pre-processing process before various image applications. It aims to effectively remove the noise from the original image, and while retain the details, structures and edges of the original image as much as possible [1]-[5]. Due to the advantages of the multi-resolution and edge detection characteristics, wavelet transform is widely used in various signal processing such as feature extraction, signal compression, image denoising, *etc.* [1]. In the wavelet domain, the current image denoising method using global threshold does not take the local characteristics of the wavelet coefficients into account. Because the image localization method in the spatial domain cannot adapt to the directional characteristics of the wavelet coefficients, the results of which is not ideal in the wavelet domain [6]-[10]. [11] proposed Local Wiener Filtering with Directional Windows (LWFDW), which is a localized wavelet threshold denoising method based on elliptic directional windows. In the wavelet coefficient matrix, local elliptic windows are set along the main feature direction, and the wavelet coefficients in each local elliptic window will have a larger probability of a higher similarity. This kind of local elliptic window can better adapt to the wavelet coefficients consistent with the main feature direction. However, without any methods applied to protect the edge information of image, the denoising method in [11] will result in unclear edge and pseudo-Gibbs phenomena in the recovery image. For addressing the mentioned-above problem, we propose an adaptive wavelet threshold denoising method based on elliptic directional window with edge detection to protect the wavelet coefficients, and further improve the denoised performance of the restored image.

II. WAVELET THRESHOLD DENOISING

The threshold denoising model is expressed as:

$$\mathrm{y} = \bar{\mathrm{y}} + \mathrm{n} \tag{1}$$

where, y represents the noisy image, $\bar{y}$ is the clear image, and n stands for two-dimensional Gaussian noise. The model in wavelet domain can be gotten as follows:

$$W_y = W_{\bar{y}} + W_n \tag{2}$$

where, W_y, $W_{\bar{y}}$ and W_n represent the wavelet coefficient of the noisy image, the clear image and the noise, respectively.

There are many commonly used thresholds estimation methods in wavelet domain, such as the general threshold VisuShrink [12], the SUREShrink threshold [6], the Minimaxi threshold [14], GCV threshold [15] and Bayesian threshold (BayesShrink) [8] and *etc.*. In this paper, the BayesShrink threshold estimation method with the best denoised performance is adopted, which is reviewed as follows.

First, the Bayesian risk function is denoted as:

$$r(T) = E\left(\widehat{W}_y - W_y\right)^2 = E_{W_y} E_{W_y|W_{\bar{y}}}\left(\widehat{W}_y - W_y\right)^2 \tag{3}$$

where, $\widehat{W}_y$ is the estimated coefficient after thresholding. $W_{\bar{y}}$ obeys the generalized Gaussian distribution with parameters $(\sigma_{\bar{y}}, \beta)$, where $\sigma_{\bar{y}}$ is the standard deviation of wavelet coefficients of the clear image. And β is the shape parameter. W_y obeys the Gaussian distribution with the parameter $(W_{\bar{y}}, \sigma_n^2)$ under the condition of variable $W_{\bar{y}}$, that is, $W_y|W_{\bar{y}} \propto (W_{\bar{y}}, \sigma_n^2)$, where σ_n^2 is the variance of the noisy wavelet coefficient. Minimize the risk function $r(T)$ to get the optimal threshold T_{Bayes}. Numerical experiments show that the threshold T_{Bayes} is proportional to σ_n^2 and inversely proportional to $\sigma_{\bar{y}}$, namely [8]:

$$T_{Bayes} = \frac{\sigma_n^2}{\sigma_{\bar{y}}} \tag{4}$$

In the wavelet threshold denoising theory, it is necessary to determine the threshold function to quantize the wavelet coefficients. Commonly used threshold functions are hard threshold function, soft threshold function and semi-soft threshold function and *etc.* [6,9,12,13]. After wavelet transformation, useful signals are concentrated on scale coefficients and a small number of wavelet coefficients with larger amplitudes, while the energy of noise is dispersed in all coefficients with small amplitude. So, the wavelet coefficients smaller than a certain threshold are set to zero, and the wavelet coefficients larger than the threshold are reduced by the threshold shrinkage, which can achieve noise reduction with principle of the minimum mean square error. The reason for the use of the soft threshold function in this paper is to avoid "one size fits all" impact of the hard threshold. That is, the wavelet coefficients larger than T_{Bayes} are uniformly subtracted from T_{Bayes}, and those smaller than $-T_{Bayes}$ are uniformly added to T_{Bayes}. The image restored by the soft threshold function will be visually smoother.

III. WAVELET COEFFICIENT LOCALIZATION METHOD BASED ON ELLIPTIC WINDOW

Compared with the global threshold denoising method, the local threshold denoising method can better retain the local features of the image. The existing methods for image localization are composed of region merging method [16], elliptic window sampling method [11] block matching method [17], strict sampling method using fixed-size square windows [18] and *etc.*. Since better adapting to the main characteristic direction of the wavelet coefficients, the local elliptic window is adopted to compute the wavelet coefficients for the localization method. After the image undergoes wavelet transformation, three wavelet coefficient matrices are generated at each decomposition scale. Each coefficient matrix contains different directional characteristics, which are horizontal, vertical, and diagonal directions. Thus, setting local elliptic windows in each coefficient matrix along its main characteristic direction will make the wavelet coefficients in each local window more likely to have higher similarity [11]. The definition of the elliptic window is denoted by

$$W(r,a) = \{(m,n) \in Z^2 : m^2 + a^4 n^2 \le a^2 r^2\} \tag{5}$$

$$W_{\mathrm{d}}(\mathrm{r},\mathrm{a}) = \{(\mathrm{m},\mathrm{n}) : \min\{\mathrm{a}^4\mathrm{m}^2 + \mathrm{n}^2, \mathrm{a}^4\mathrm{n}^2 + \mathrm{m}^2\} \le \mathrm{a}^2\mathrm{r}^2\} \tag{6}$$

where, r and a stand for the direction parameters of the elliptic window; (m,n) represent the coordinate index of the pixel. Eq. (5) represents an elliptic window in the horizontal and vertical directions, and Eq. (6) represents an elliptic window in the diagonal direction. The shapes of the elliptic window corresponding to $W(6,2)$, $W(6,1/2)$, $W_d(6,2)$ are shown in Fig. 1.

(a) (b) (c)

Fig. 1 Illustration of elliptical directional windows. (a)horizontal;(b)vertical;(c)diagonal

IV. THE PROPOSED METHODOLOGY

The local elliptic window consistent with the main feature direction of the wavelet coefficients can better adapt to the wavelet coefficient features. If protective measures can be taken to preserve the characteristics of the image, the noise reduction effect could be further improved. To this end, this paper proposes a local adaptive wavelet threshold denoising method based on elliptic directional window and edge detection. Firstly, the image is decomposed by wavelet, and its wavelet coefficient is edge-detected. And then, the wavelet coefficients are sampled by the elliptic directional window. During sampling, some elliptic windows may contain some wavelet coefficients with larger amplitudes that represent edge features. In order to protect these wavelet coefficients, the threshold amplitude is reduced by multiplying a weight c less than 1 when the wavelet threshold is calculated in elliptic windows. For a more accurate result, the iteration is carried out to update the local signal standard deviation estimation. The detailed steps are introduced as follows:

1) A J-scale wavelet transform of the noisy image is performed to obtain scale coefficients $A_j (j = 1, \ldots, J)$, and wavelet coefficients $W_{j,k} (j = 1, \ldots J, k = h, v, d)$, where h, v, d represent the horizontal, vertical and diagonal directions, respectively.
2) The edge detection of the wavelet coefficient image with each scale and each direction are performed to obtain the wavelet coefficients of the edges.
3) The wavelet coefficients with each scale and each direction are locally sampled by an elliptic window, and then the local threshold of the wavelet coefficients in the window is estimated according to Eq (7).

$$T_{j,k,m,n} = \frac{\hat{\sigma}_{n\,j,k}^2}{\hat{\sigma}_{y_{j,k,m,n}}} \tag{7}$$

where $T_{j,k,m,n}$ is the threshold value of wavelet coefficient at scale j, direction k, and position (m,n). If the elliptic window contains wavelet coefficients representing the edge, $T_{j,k,m,n}$ can be recast as

$$T_{j,k,m,n} = c\frac{\hat{\sigma}^2_{n_{j,k}}}{\hat{\sigma}_{y_{j,k,m,n}}} \tag{8}$$

among them, $\hat{\sigma}_{n_{j,k}}$ represents the noise standard deviation estimation of the wavelet coefficient, and the estimation formula of which is

$$\hat{\sigma}_{n_{j,k}} = \frac{median(|W_{j,k}|)}{0.6745} \tag{9}$$

where $W_{j,k}$ corresponding to scale j and direction k. $\hat{\sigma}_{y_{j,k,m,n}}$ represents the local signal standard deviation of the wavelet coefficients in the corresponding elliptic window, and its estimation formula is defined as:

$$\hat{\sigma}_{y_{j,k,m,n}} = \sqrt{\left(\frac{1}{N_R}\sum_{(p.q\epsilon R)} W^2(m+p,n+q) - \hat{\sigma}^2_{n_{j,k}}\right)_+} \tag{10}$$

where, R represents the elliptic window, N_R stands for the number of wavelet coefficients in the elliptic window. Function $(a)_+$represents

$$(a)_+ = \begin{cases} 0, a \le 0 \\ a, otherwise \end{cases} \tag{11}$$

4) The soft threshold function processing of each wavelet coefficient needs to be performed. The estimated wavelet coefficients $\widehat{W}_{j,k,m,n}$ at scale j, direction k, and position (m,n) can be obtained as

$$\widehat{W}_{j,k,m,n} = \begin{cases} 0, |W_{j,k,m,n}| \le T_{j,k,m,n} \\ sgn(W_{j,k,m,n})(|W_{j,k,m,n}| - T_{j,k,m,n}), |W_{j,k,m,n}| > T_{j,k,m,n} \end{cases} \tag{12}$$

5) The wavelet coefficients quantized by thresholds and scale coefficients are used to implement the inversed wavelet transform to obtain the restored image after the first noise reduction. Repeat steps 1) ~3) for the coarsely restored image to obtain new wavelet coefficients $W'_{j,k}(j = 1,\ldots J, k = h,v,d)$, and update the local signal standard deviation of the wavelet coefficients in each elliptic window $\sigma'_{y_{j,k,m,n}}$

$$\sigma'_{y_{j,k,m,n}} = \sqrt{\frac{1}{N_R}\sum_{(p,q\epsilon R)} W'^2(m+p,n+q)} \tag{13}$$

As other steps and data remain unchanged, and new thresholded wavelet coefficients are obtained.

6) Finally, the inverse wavelet transformation on scale coefficients and new thresholded wavelet coefficients is implemented to obtain denoised images. The flowchart of details is shown in Fig. 2.

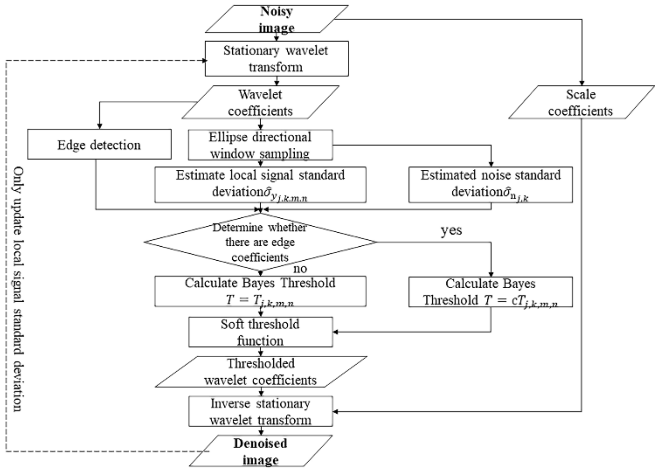


Fig. 2 Flow chart of wavelet local adaptive threshold denoising method based on elliptic directional window and edge detection

V. Experiment Results and Analysis

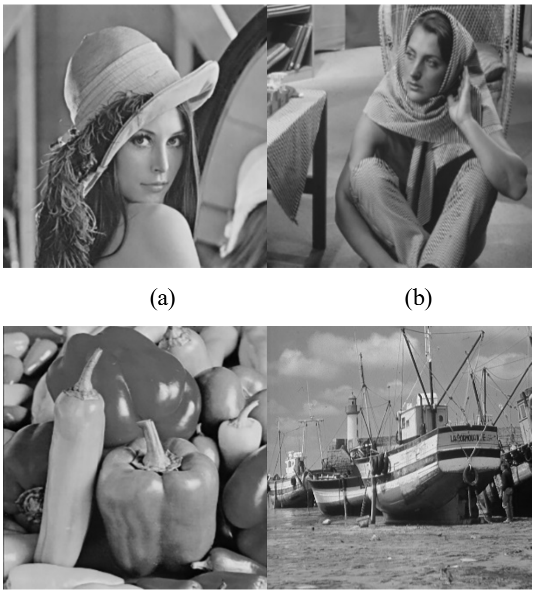
In order to test the effectiveness of the denoising method proposed in this article, with the platform of MATLAB2018, four standard gray-scale test images-Lena, Barbara, Peppers, and Boats are selected for comparison experiments. The size of the images is 512×512. This paper verifies the importance of edge detection before threshold processing. The selected reference method is based on the LWFDW method [11]. The Symlet-8 wavelet base of the stationary wavelet transform is selected, the decomposition scale of which is 4. The edge detection operator selects the Canny operator, the parameters of the elliptic window are$W(6,2)$,$W(6,1/2)$,$W_d(6,2)$, and the weight c is set to 0.9. The noise standard deviations σ used in the simulation are set to 10, 15, 20, 25, and 30, respectively. Peak signal-to-noise ratio (PSNR) [8] and structural similarity (SSIM) [17] are chosen to evaluate the quality of denoised results.

The numerical results of the experiment are shown in Table 1. The bold items represent the best results among the proposed method and the reference method under the same experimental conditions. Fig. 3 and 4 respectively show the original image and the noisy image with σ=20. The results in Table 1 show that (1) when the noise level is low, the numerical indexes of the two local wavelet threshold noise reduction methods are close. (2) When the noise level is high, the proposed method is better than the LWFDW method, and the higher the noise level, the more obviously the proposed method is better than LWFDW. The results in Fig. 5 show that the two local wavelet threshold methods have similar recovery capabilities to edges, but the proposed method is visually smoother than the LWFDW, and has fewer pseudo-Gibbs phenomena.

TABLE I. Quality Evaluation Indexes of Four Test Images and Three Wavelet Denoising Methods

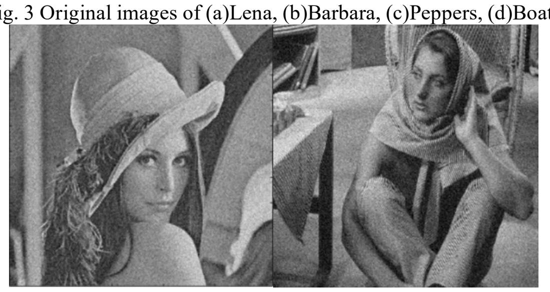
Test image	Noise standard deviation	PSNR		SSIM	
		LWFDW	Proposed	LWFDW	Proposed
Lena	σ=10	35.63	**35.80**	0.92	**0.93**
	σ=15	33.42	**33.77**	0.89	**0.90**

	σ=20	31.79	**32.32**	**0.88**	0.85
	σ=25	30.51	**31.18**	0.81	**0.86**
	σ=30	29.45	**30.27**	0.77	**0.84**
Barbara	σ=10	33.78	**33.78**	**0.93**	**0.93**
	σ=15	31.33	**31.39**	0.88	**0.89**
	σ=20	29.65	**29.71**	0.85	**0.86**
	σ=25	28.32	**28.47**	0.81	**0.83**
	σ=30	27.21	**27.39**	0.77	**0.79**
Peppers	σ=10	35.12	**35.17**	**0.90**	**0.90**
	σ=15	33.20	**33.48**	0.87	**0.88**
	σ=20	31.72	**32.18**	0.83	**0.86**
	σ=25	30.46	**31.07**	0.80	**0.84**
	σ=30	29.50	**30.28**	0.76	**0.82**
Boats	σ=10	**33.24**	33.15	**0.87**	**0.87**
	σ=15	31.25	**31.28**	**0.83**	**0.83**
	σ=20	29.80	**29.96**	0.79	**0.80**
	σ=25	28.67	**28.95**	0.75	**0.77**
	σ=30	27.73	**28.08**	0.71	**0.74**

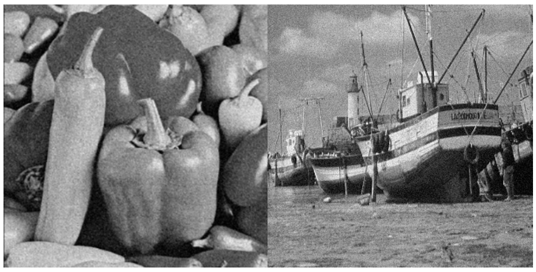


(a) (b) (c) (d)

Fig. 3 Original images of (a)Lena, (b)Barbara, (c)Peppers, (d)Boats



(a) (b)



(c) (d)

Fig. 4 Noisy images of (a)Lena, (b)Barbara, (c)Peppers, (d)Boats

(a) (b)

Fig. 5 Comparison of the noise reduction results of Lena, Barbara, Peppers and Boats at the noise level. (a) LWFDW (b) The proposed denoising method

VI. Conclusion

This paper proposes a local adaptive wavelet threshold denoising method based on elliptic directional window and edge detection, aiming to protect the edge features and improve the denoised results. The method first performs wavelet transformation for the image, and then applies edge detection to the wavelet coefficient of the image. Next, wavelet threshold estimation on the wavelet coefficients inside the window is computed after sampling the wavelet coefficients of image by the elliptic window. Owing to the elliptic windows which contains the wavelet coefficients representing the edges, we add a weight less than 1 to the restored image after the first noise reduction to reduce the threshold shrinkage to protect the edge. At the same time, in order to obtain a more accurate estimation, the iteration is applied to update the estimated value of the local signal standard deviation to obtain a new thresholded wavelet coefficient. Finally, the denoised image can be achieved by the inverse wavelet transformation with the scale coefficient and the denoised wavelet coefficients. By comparison with LWFDW, experimental results show that the proposed method can achieve better denoising effects in terms of numerical indicators, and the denoised image is smoother in the uniform area and has fewer pseudo-Gibbs phenomena in visual.

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