Heat Source Size Effect on Heat Transfer in a Local Heated From Below at High Rayleigh Number

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January 24, 2020

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Abstract. In this work we used the Lattice Boltzmann method (LBM) to model the turbulent flow in a local filled with air, of side H, and heated from below by a constant temperature θ_C=1. One of the vertical walls has a cold portion of temperature θ_F=0. The other walls are adiabatic. The calculations were performed in two dimensions (2D) for Rayleigh numbers $Ra=10^8$ and $Ra=1.5\times10^8$ and for different size ratio of the heat source ($0.2 \leq Lr = \ell / H \leq 0.8$). The results are presented in the form of streamlines, isotherms and temperature evolution. The heat transfer is studied in terms of the average Nusselt number calculated on the hot wall. Results show that the temperature at the center and the heat transfer increases by increasing the *Lr* or *Ra*.

Keywords: *Lattice Boltzmann method, Turbulent natural convection, Local heated from below, High Rayleigh number.*

I. INTRODUCTION

During the last decades, an increasing interest has been given to thermal transfers by natural convection, because of these applications in several fields of research and engineering (heating and cooling of premises, heating of electrical circuits, nuclear power station, etc…). Several authors have studied and simulated turbulent or laminar natural (or mixed) convection using numerical methods: Finite Difference Method (FDM), Finite Volume Method (FVM), Lattice Boltzmann Method (LBM) etc…. Among these authors De Vahl Davis [1] studied and imposed a reference solution of natural convection in a square cavity filled with air and differentially heated. Le Quéré [2] studied the same configuration as De Vahl Davis using a more precise method for high Rayleigh numbers. In recent years several researchers [3-8] used the lattice Boltzmann method for the simulation of natural or mixed convection in turbulent regime. Mohamad [9] established the various stages and the steps necessary for the application of the method of Lattice Boltzmann (LBM), and made a comparative study between the results obtained by the LBM method and the finite difference method (MDF). The results found by the two methods are in good agreement in different applications. Dixit et al. [10], in turn, used the LBM method to simulate convection flows in a square cavity for high Rayleigh numbers and for this they did a good validation and comparison in the two types of laminar and turbulent convection by the use of an interpolation to complete Lattice Boltzmann. Abouricha et al. [11,12] used LBM for the simulation of turbulent natural convection in a large-scale square cavity and for high Rayleigh numbers. Results were presented for different values of Rayleigh numbers in the form of streamlines and isotherms as well as the evolution of temperature and velocity in the median plane of the cavity. The global local Nusselt number is also presented. The authors found a correlation for the heat transfer characterized by the Nusselt number as a function of the Rayleigh number.

Our contribution, consists in the characterization of turbulent natural convection flows in a closed room heated from below. We use the LBM numerical method for this purpose. We value the influence of the size of the heat source Lr and the Rayleigh Ra number on the heat transfer.

II. PHYSICAL PROBLEM AND METHOD OF SOLUTION

1. Physical Problem

The configuration studied is a square room in 2D of side H (Fig. 1) heated from below by a temperature assumed to be constant $\theta_c = 1$. One of the vertical walls is provided with a cold portion $\theta_F = 0$ simulating a glass door. The other walls are adiabatic. The variation of the density is subject to the Boussinesq approximation. The cavity is filled with air (Pr = 0.71).

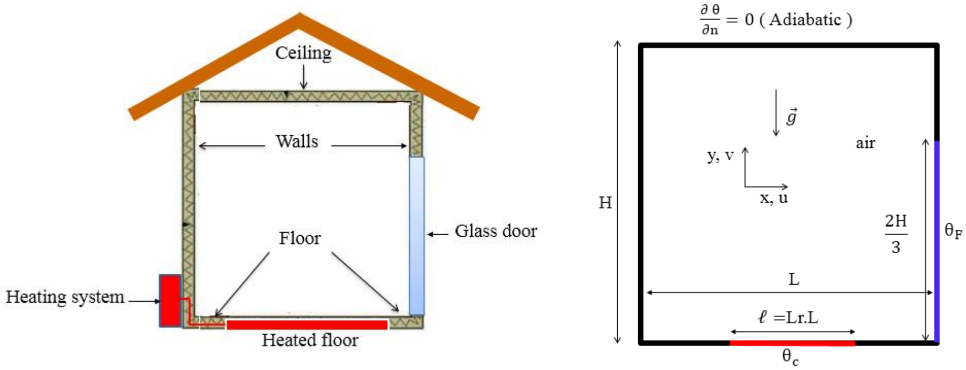

FIGURE 1. *Physical problem and 2D configuration*

2. Lattice Boltzmann Method (LBM)

The LB model used here is the same as that employed in [9,11]. This model utilizes two single particle distribution functions, the first $f_k(x,t)$for dynamic and the second $g_k(x,t)$, for thermal field simulations, respectively. So we consider the model LBM called model D2Q9 two-dimensional and nine discrete velocities (Fig. 2).

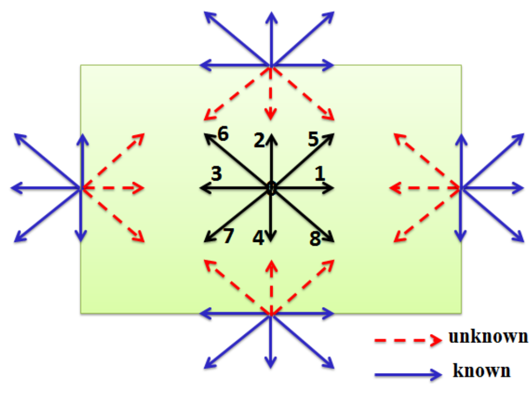

FIGURE 2. *D2Q9 model*

The BGK approximation LB equation without external forces can be written as:

$$\frac{\partial f_k}{\partial t} + c_k \frac{\partial f_k}{\partial x} = \Omega(f_k) \qquad (1)$$

where f_k are the particle distribution functions defined for the finite set of discrete particle velocity vectors c_k, x is the position and t is the time . The collision operator $\Omega(f_k)$, on the right hand side of Eq. (1) uses the so called Bhatangar-Gross-Krook (BGK) approximation [13]. For single time relaxation, LB approximation is that the collision term $\Omega(f_k)$ will be replaced by:

$$\Omega(f_k) = -\frac{1}{\tau_m}(f_k - f_k{}^{eq}) \qquad (2)$$

Where τ_m is the relaxation time for the flow and $f_k{}^{eq}$ is the local equilibrium distribution functions that have appropriately prescribed functional dependence on the local hydrodynamic properties. The equilibrium distribution can be formulated as in [11]:

$$f_k^{eq}(x,t) = \omega_k \rho(x,t)\left(1 + 3\frac{c_k u}{c^2} + \frac{9}{2}\frac{(c_k\, u)^2}{c^4} - \frac{3}{2}\frac{u^2}{c^2}\right) \qquad (3)$$

where u and ρ are the macroscopic velocity and density, respectively, and the ω_k are the weights factors and c_k the discrete velocities that are given for D2Q9 by:
$\omega_\mathrm{k} = \frac{4}{9}$ for k = 0, $\omega_\mathrm{k} = \frac{1}{9}$ for k = 1 ... 4, $\omega_\mathrm{k} = \frac{1}{36}$ for k = 5 ... 8.
$c_k = (0\,,0\,)$ for k = 0, $c_k = (\pm 1\,,0\,)\,;\ (0\,,\pm 1\,)$ for k = 1 ... 4, $c_k = (\pm 1\,,\pm 1\,)$ for k = 5 ... 8.

where $c_k = \frac{\Delta x}{\Delta t}$, Δx and Δt are the lattice space and the lattice time step sizes, respectively, which are set to unity.
Finally The BGK approximation lattice Boltzmann equation with external forces can be written as for the flow field:

$$f_k(x + c_k\Delta t\,, t + \Delta t) = (1 - w_m) f_k(x\,,t) + w_m f_k^{eq}(x\,,t) + \Delta t\, F_k \qquad (4)$$

where $w_m = \frac{\Delta t}{\tau_m}$. For momentum w_m is prescribed through kinematic viscosity as:

$$w_m = \frac{1}{3\upsilon + 0.5} \qquad (5)$$

F_k is an external force term. The Boussinesq approximation is applied to the buoyancy force term. In this case the external force F_k appearing in Eq. (8) is given by:

$$F_k = 3\omega_k \frac{\rho g \beta \Delta T . c_k}{c^2} \qquad (6)$$

where g is the gravitational vector, ρ is the density, ΔT is the temperature difference between hot and cold boundaries and β is the thermal expansion coefficient.
Finally, the basic hydrodynamic quantities, such as density ρ and velocity u, are obtained through moment summations in the velocity space:

$$\rho(x,t) = \sum_{k=0}^{k=8} f_k(x,t) \qquad (7)$$

$$\rho u(x,t) = \sum_{k=0}^{k=8} c_k f_k(x,t) \qquad (8)$$

For scalar function temperature or energy, other distribution functions are defined as,

$$g_k(x + c_k\Delta t\,, t + \Delta t) = (1 - w_s) g_k(x\,,t) + w_s g_k^{eq}(x\,,t) \qquad (9)$$

where $w_s = \frac{\Delta t}{\tau_s}$ is related to diffusion coefficient as:

$$w_s = \frac{1}{3\alpha + 0.5} \qquad (10)$$

where τ_s is the relaxation time for temperature and α is the thermal diffusion coefficient. The equilibrium distribution functions for the temperature field, i.e. Eq. (13), can be used at first-order.

$$g_k^{eq} = \omega_k T(x,t)\left[1 + 3\frac{c_k u}{c^2}\right] \tag{11}$$

The temperature *T(x,t)* is calculated by:

$$T(x,t) = \sum_0^8 g_k(x,t) \tag{12}$$

3. Boundary conditions

The boundary conditions associated with the problem are as below:

- **Dynamic boundary conditions:**

$$u = v = 0 \text{ For } x = (0,1) \text{ and } 0 \le y \le 1$$
$$u = v = 0 \text{ For } y = (0,1) \text{ and } 0 \le x \le 1$$

- **Thermal boundary conditions:**

$$\theta = \theta_c = 0 \quad \text{For} \quad x = 1 \text{ and } 0 \le y \le \frac{2}{3}$$
$$\theta = \theta_h = 1 \quad \text{For} \quad y = 0 \text{ and } \frac{1-Lr}{2} \le x \le \frac{1+Lr}{2}$$

$\frac{\partial \theta}{\partial n} = 0$ for adiabatic walls, n is the normal of walls. where $\theta = \frac{T - T_c}{T_h - T_c}$ is the dimensionless temperature.

- **« bounce-back » conditions for the imposed temperature:**

$$g_k(x,t) = \left(\omega_k + \omega_{opp(k)}\right)\theta - g_{opp(k)}(x,t) \tag{13}$$

4. Nusselt number calculation

The heat transfer is examined by calculation the average Nusselt numbers on the hot wall using this integration follow:

$$\overline{Nu} = \frac{-1}{Lr}\int_{\frac{1-Lr}{2}}^{\frac{1+Lr}{2}} \left.\frac{\partial \theta}{\partial y}\right|_{y=0} dx \tag{14}$$

III. VALIDATION OF NUMERICAL METHOD

The model was validated by considering a differentially heated square cavity containing air of Prandtl number Pr=0.71 . This comparison was made for a value of the Rayleigh number Ra = 10^8. The numerical results are in good agreement with the results of the reference [2], (table 1).

TABLE 1. *Comparison of the results.*

	Nos résultats *Ra=10^8*	Le Quéré [2] *Ra=10^8*	Deviation
V_{max}	2156.52	2222.39	2.9%
X	0.011	0.012	
$\overline{Nu}$	30.79	30.225	1.8%

IV. RESULTS AND DISCUSSION

1. Streamlines and isotherms

Fig. 3 represents the isothermal lines and the current lines for Ra = 1.5×10^8 and for Lr = 0.8. The isotherms are more intense in the vicinity of the active portions where we are witnessing the development of a thermal boundary layer. At the heart of the room, the isotherms become distorted and form closed isotherms representing the puffs of warm air rising upwards. The flow is formed by a main cell which rotates clockwise with small secondary cells located at (X = 0.2; Y = 0.85) and (X = 0.95; Y = 0.75) and turning in the opposite direction.

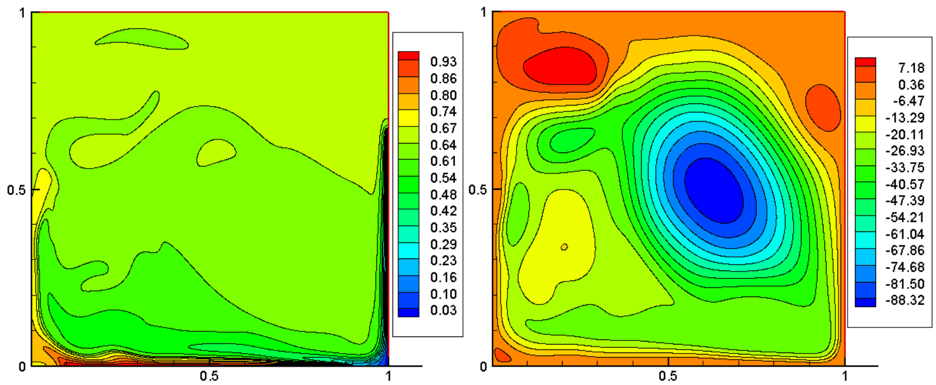

FIGURE 3. *Isotherms (Left) and streamlines (right) for Lr=0.8 and Ra=1.5×10^8.*

2. Temperature profile

The temporal evolution of the temperature θ *(t)* at the center of the room is shown in Fig.4 for a size of the heat source varying in the range 0.2≤Lr≤0.8 and for Ra = 10^8. Generally the evolution of the temperature represents a transition towards a permanent value of the temperature θp which varies linearly with Lr in the form:

$$\theta p(Lr) \approx 0.54 \times Lr + 0.2 \qquad (15)$$

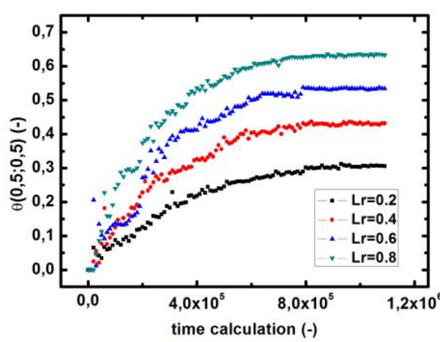

FIGURE 4. *Time evolution of the temperature at the center for different values of Lr and for Ra=10^8.*

3. Heat transfer

The heat transfer is examined by calculating the mean Nusselt number $\overline{Nu}$. From Table 2 we notice that $\overline{Nu}$ increases with the increase in Ra and with that of Lr.

TABLE 2. *Values of mean Nusselt number $\overline{Nu}$ for different values of Ra and of Lr.*

Lr	***Ra=10^8^***	***Ra=1.5×10^8^***
0.2	8	---
0.4	12.5	---
0.6	16	---
0.8	20	23

V. CONCLUSION

We carried out this numerical study to developing a house code based on LBM which allowed us to treat the natural convection flows in laminar or turbulent regime. The objective is to study the heat transfer by convection in a large-scale square cavity heated from below for high Rayleigh number. The results show that:

- The flow is turbulent similar to the Rayleigh-Bénard type.
- The temperature in the center increases with increasing of Lr. And can be correlated in this game of Lr by:

$$\theta p(Lr) \approx 0.54 \times Lr + 0.2$$

- The heat transfer is accentuated by the increase in Ra or Lr.

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