Select and Accept

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Abstract

The aim of this paper is to present how to solve the problem of selecting a candidate up to his or her acceptance trough game theory. The originality this paper proposes is how this problem will be approached. It will be treated as a single game which is made up of two parts, going as far as to state that the payoffs in the first part of the game will be the mediators of the second part of the game. In order to represent the problem described above, a new form of game representation will be used – code form – consisting of a table which contains all of the information, without any suppression or “adulteration “with regards to the game.

Keywords Game Theory, Two-part Game, Code Form

1. Introduction

The game we describe represents a real life situation: candidate selection for a given job. We will show, because of the complexity of the relationships between different parties and the fact that each one of them is an essential element in the game’s decision-taking, that the best candidate is not always the best option. On the other hand, through this game we also show that knowing the fundamental elements that constitute a game - who are the players, what are the strategies for each player and the payoffs each player can receive - is not enough to know its solution, if one even exists.

2. The Game

A presidential decree reduced the number of candidates to the vice-presidency to three people. Each of the three candidates are ranked on a scale from 1 (lower) to 10 (higher).The presidential board attributed 10 points, 8 points and 5 points to the candidate classified in 1st place, 2nd place and 3rd place respectively. The probabilities of candidate i (i=1,2,3) accepting the j-th offer to run for the vice-presidency have been defined, considering that the first j-1 offers to the others have been declined, are denoted by pij where:

$$p11 = 0{,}5 \quad p12 = 0{,}2 \quad p13 = 0$$
$$p21 = 0{,}9 \quad p22 = 0{,}5 \quad p23 = 0{,}2$$
$$p31 = 1 \quad p32 = 0{,}8 \quad p33 = 0{,}4$$

What we want to know is what is the order in which the three potential candidates be offered the vice-presidential nomination if the presidential decree maximizes the expected number of points, supposing that no candidate is requested more than once and, each time a candidate rejects, another one is requested, until at least one has accepted or all have rejected.

3. Game Elements

This game which is made up of two parts – a selection process and an acceptance process – attests that the payoffs in the first part of the game (potential candidates) will be the intermediaries of the (decision elements). Thus, we have:

Players:
Presidential Board:4
Potential Candidates:
Player classified in 1st place –1;
Player classified in 2nd place – 2;
Player classified in 3rd place – 3

Strategies:

Presidential Board:
The presidential board wants to establish the order in which the potential candidates will be invited to maximize the expected number of points. In this way the strategy for the presidential board will be the order in which the three potential candidates can be offered the vice-presidential nomination until at least one has accepted or all have rejected the offer - P.

Potential Candidates:
The strategy of each potential candidate is to accept the offer – A - or to reject the offer - R.

Payoffs:

Presidential Board:
The presidential board payoff is (a function of the attributed points in the daily preselection and of the potential candidates' probabilities of acceptance of the vice-presidency) the expected number of points of each possibility in the order of the proposal presented to the potential candidates. Thus, for instance, we have: Player 1 rejects the offer, 3 rejects the offer, 2 accepts the offer, presidential board payoff is:

$$\frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times 8 = 0.16 ,$$

where $\frac{1}{2}$ is the probability of player 1 rejecting the offer, $\frac{1}{5}$ the probability of player 3 rejecting the offer and $\frac{1}{5}$ the probability of player 2 accepting the offer.

Potential Candidates:

For these players it is possible to define: if the player accepts the offer, he gets the "total prize", that is, he gets payoff 1. On the other hand, if he rejects the proposal, he does not get anything so his payoff will be 0.

4. Game Representation

To represent the game, code form game representation will be used. Code form is basically a table where the strategies that are available to any player are codified.

Definition: A code form game consists of a finite table, with evident extension in the case of infinite moves and infinite players, where only some cells are filled. The cells will be filled in the same order that the game is played. For that it is needed:

$R = \{1,2,\ldots,R\}$ - a set of rounds;

$J = \{1,2,\ldots,J\}$ - a set of moves;

$C = \{1,2,\ldots,J+3\}$ - a set of columns;

$L = \{1,2,\ldots,L\}$ - a set of lines;

$N = \{1,2,\ldots,N\}$ - a set of players;

$E_N = \{e_1, e_2, \ldots, e_N\}$ - a set of strategies available to each player;

$E = E_1 \times E_2 \times \ldots \times E_N$ - a set of all such strategy profiles - space of strategies profiles;

$\begin{array}{l} RN : L \times \{1\} \to R \\ \quad a_{i1} \to r \end{array}$, a function that indicates the round number;

$\begin{array}{l} PN : L \times \{2\} \to J \\ \quad a_{i2} \to j \end{array}$, a function that indicates the move number;

$$JE : L \times C \to N \times E_N, \quad c \neq 1,2, J+3, \quad a_{ic} \to (N, e_N)$$

a function that indicates who moves and what action is played;

$$PJ : L \times \{J+3\} \to IR^N, \quad a_{i,J+3} \to (u_1, u_2, ..., u_N)$$

a function that gives the payoff of every player (for all the players) where $u_N : E \to IR$ is a von Neumann-Morgenstern utility function.

Note: It can be denoted that when $RN(a_{i1}) = RN(a_{i-1,1})$, $PN(a_{i2}) = PN(a_{i-1,2})$ and $JE(a_{ic}) = JE(a_{i-1,c})$, the cells are not filled. The line is changed when the move changes. The column is changed when the player changes.

Code form game idea lies in the game estimated linear reading. The table is built containing the whole game information. The table 1 illustrates the code form game representation in the game example that is considered.

Reading from left to the right, the first table column indicates the period number and the second column indicates the move number. The following columns mention who moves when and in what circumstances and what action is played when somebody is called upon to move. Last column indicates the payoffs vector in accordance with the strategies chosen by the players.
As we can see, it is easy to verify that the order in which the three potential candidates can be offered the vice-presidential nomination must be:
To invite in the first place the candidate classified in 2nd place, 2; if he rejects the proposal the candidate classified in first place, 1, should be invited and if he does not accept the candidate classified in third place, 3 should be invited.

Table 1 Code Form Game

1	1	(4, P)				
1	2		(1, A,0.5)			(5,1,0,0)
			(1, R,0.5)			
	3			(2, A,0.5)		(2,0,1,0)
				(2, R,0.5)		
	4				(3, A,0.4)	(0.5,0,0,1)
					(3, R,0.6)	(0,0,0,0)
	3			(3, A,0.8)		(2,0,0,1)
				(3, R,0.2)		
	4				(2, A,0.2)	(0.16,0,1,0)
					(2, R,0.8)	(0,0,0,0)
	2		(2, A,0.9)			(**7.2**,0,1,0)
			(2, R,0.1)			
	3			(1, A,0.2)		(0.2,1,0,0)
				(1, R,0.8)		
	4				(3, A,0.4)	(0.16,0,0,1)
					(3, R,0.6)	(0,0,0,0)
	3			(3, A,0.8)		(**0.4**,0,0,1)
				(3, R,0.2)		
	4				(1, A,0)	(**0**,1,0,0)
					(1, R,1)	(**0**,0,0,0)
	2		(3, A,1)			(5,0,0,1)
			(3, R,0)			
	3			(1, A,0.2)		(0,1,0,0)
				(1, R,0.8)		
	4				(2, A,0.2)	(0,0,1,0)
					(2, R,0.8)	(0,0,0,0)
	3			(2, A,0.5)		(0,0,1,0)
				(2, R,0.5)		
	4				(1, A,0)	(0,1,0,0)
					(1, R,1)	(0,0,0,0)

5. Conclusion

As we could see, the analyzed game illustrates that the player who has decision power is not always the one who decides the game. In other words, the presidential board is the player who dominates the situation; therefore, he decides who the best candidate is, but, in fact, the ones who actually decide the game are the potential candidates when they accept or they refuse the proposal. In fact, the potential candidates are the ones who determine the game's outcome; starting from a situation of weakness they gain control of the game. They are the result of the first part of the game and become the deciding elements of the second part of the game.
On the other hand, this game shows how much the Game Theory is still a science with a long way to go for games with more than two players. No solution concept for these kinds of games is universally accepted. One reason for this is the situation described herein since we could not find the respective equilibrium using existing solution concepts, such as the Nash equilibrium, the Shapley value, and so on. This in no way reduces the significance of Game Theory. In fact, besides motivating the theorists and considering the already developed results (we cannot
forget Game Theory is one of the few theories that defines rational procedures in what were previously considered irrational situations and that its concepts and ideas have already very provided important and deep knowledge in the formulation of situations to real world conflicts), we can conclude how much Game Theory is an asset to humanity.

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