A Luenberger-like State Estimation with False Data Injection on Input and Model Mismatch: LMI Approach

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July 5, 2022

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Abstract

This paper investigates a proportional-integral Luenberger-like state estimation for continous linear systems under disturbed inputs or false data inhection attacks on inputs and model mismatch. A general design algorithm is proposed to estimate the states of a linear system in the presence of model mismatch, external disturbance, and disturbed inputs / false data injection. In reality, the absence of the conditions is very rare, therefore it is required to develop a method for designing robust observers in presence of uncertainties, external disturbances, and disturbed inputs. Lyapunov method and LMI theory have been used to obtain the gains of the Proportional-Integral Observer (PIO). The stability of the proposed PI observer is proved and with a numerical example, its efficiencies have been shown under different cases including model mismatch, disturbances, and attacks. The results illustrate that the proposed algorithm can estimate the state of a system according to defined conditions.

***Keywords*:** Luenberger-like observer, Linear Matrix Inequality (LMI), Lyapunov Stability Theory, Model Mismatch, False Data Injection.

1 Introduction

Modern physical systems are autonomous engineering systems embedding physical components, communications, and computational capabilities/control logic [1]. Recent advances in modern engineering i.e., power grids [2-3] lead many researchers to consider various problems in this field such as stability analysis [4], fault detection [5], and security problems [6]. Tight coupling of information technology and physical component made modern systems vulnerable to malicious attacks and disturbing signals [7]. Therefore, it is crucial to enhance the security of systems by novel methods such as analysis of false-data injection attacks(FDI) on inputs and outputs [8], secure control [9], observer-based state identification under attacks or disturbed signals [10], and state estimation under attacks or disturbed signals[11-12].

State estimation is the problem of estimating the state when the mathematical model contains some uncertain elements. These uncertainties might be due to additive unknown noises, attack, false data injection, environmental influences, nonlinearities such as hysteresis or friction, poor plant knowledge, reduced-order models, uncertain or slowly varying parameters. In this regard, there are different strategies

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for state estimation including filter-based model [13], observer-based models [14-15], and gradient descent algorithms [16].
In modern systems, the sensors, actuators, and controllers should communicate with each other, however, in addition to sensors, actuators may be disturbed which would affect the performance of physical systems. Some discussions on disturbed actuators and actuator attacks have been presented in Fawzi et al [17]. However, the sufficient conditions for estimating under the actuator attacks have not been considered. Shoukry and Tabuada [18] developed a static optimization problem based on the Luenberger observer to estimates the state of a dynamical system in the presence of disturbing actuators and some event-triggering conditions. A secure Luenberger-like observer under spars actuators and sensors attacks presented by Yang Lu and Hong Yang [19]. The least-square technique and a new projection operator are used to reconstruct the state of systems. Sufficient conditions for the existence of observer are proposed in the terms of linear matrix inequalities (LMI). Yabin Gao et al [20] proposed a practical state estimation based on Luenberger in the delta operator framework by considering disturbance and actuator attacks. The constraints are based on linear matrix inequalities. Although the Implementation of a Luenberger observer is relatively simple; however it strongly depends on the precision of parameters and measurements [21]. All the presented studies are based on linear systems without model mismatch and disturbances. Therefore a robust observer design approach is required to guarantee the stability and accuracy of this observer in the presence of model mismatch, disturbances, and also the actuators of systems are disturbed by false data injection. In the present paper, a general method is introduced which is used for designing PI Luenberger observers in the continous linear systems with model mismatches, external disturbances, and disturbed input as false data injection. The proposed method is applicable in a wide range of problems with uncertain and nonlinear terms and also needs much less information about the system. Due to the use of Lyapunov and LMI theories, the method guarantees the stability and performance robustness of the observer.

Notation. For a matrix $M \in R^{p\times q}$, M^T defines transpose, $M > 0 (M < 0)$ defines positive(negative) definiteness, $\lambda_m(M)$ defines its smallest eigenvalue. Given a vector $v \in R^n$, $\|v\|$ is its Euclidean norm. R denotes the set of reals. $\hat{\star}$ shows the estimation of $\star$. 0 and I are zero and unit matrix with appropriate dimensions, respectively.

2 Problem Description

Let consider a MIMO LTI system which its states and output matrices contain time varying uncertainties under input false data injection as following:

$$\begin{aligned} \dot{x}(t) &= \big(A + \Delta A(t)\big)x(t) + B\big(u(t) + a_u(t)\big) \\ y(t) &= \big(C + \Delta C(t)\big)x(t) + Dw \end{aligned} \tag{1}$$

where $x \in R^n$, $y \in R^q$, $u \in R^m$, $w \in R^p$ are the state, output, input and noise disturbance vectors, respectively. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{q\times n}$, $D \in R^{q\times p}$, are known matrices. $\Delta A(t) \in R^{n\times n}$, $\Delta C(t) \in R^{q\times n}$, and $a_u(t)$ are unknown uncertainties in state and output matrices and disturbed input as a false data injection, respectiveley .

Defining z as integral of output vector:

$$z = \int_0^t y(\tau)d\tau \tag{2}$$

Then system (1) can be rewritten in the following form:

$$\begin{aligned}\dot{x}(t) &= \big(A + \Delta A(t)\big)x(t) + B\big(u(t) + a_u(t)\big)\\ \dot{z}(t) &= \big(C + \Delta C(t)\big)x(t) + Dw\end{aligned} \tag{3}$$

The matrix form of equations (3) is:

$$\begin{aligned}\dot{X} &= (A_1 + \Delta A_1)X + C_1 B\big(u(t) + a_u(t)\big) + C_2 Dw\\ Y_1 &= (C + \Delta C)C_1^T X + Dw\\ Y_2 &= z = C_2^T X\end{aligned} \tag{4}$$

Where $X \in R^{n+q}, Y_1, Y_2 \in R^q$ are augmented state, output and output integral vectors respectively. Other matrices are defined as:

$$X = \begin{bmatrix} x_{n\times 1} \\ z_{q\times 1}\end{bmatrix}, \quad A_1 = \begin{bmatrix} A_{n\times n} & 0_{q\times q} \\ C_{q\times n} & 0_{n\times q}\end{bmatrix},$$

$$\Delta A_1 = \begin{bmatrix} \Delta A_{n\times n} & 0_{q\times q} \\ \Delta C_{q\times n} & 0_{n\times q}\end{bmatrix},$$

$$C_1 = \begin{bmatrix} I_{n\times n} \\ 0_{q\times n}\end{bmatrix}, \quad C_2 = \begin{bmatrix} 0_{n\times q} \\ I_{q\times q}\end{bmatrix} \tag{5}$$

3 Luenberger-like State Estimation Design

To cancel the unwanted effects of model mismatch and disturbance, it has been proposed to use an integral action in the Luenberger observer; so called PI observer, in the following way:

$$\dot{\hat{X}} = A_1 \hat{X} + C_1 Bu + K_P(Y_1 - C\,C_1^T \hat{X}) + K_I(Y_2 - C_2^T \hat{X}) \tag{6}$$

With this observer, one can obtain the estimation error dynamics as:

$$\begin{aligned}\dot{E} &= A_1 E + \Delta A_1 X + C_2 D\,w - K_P\big(CC_1^T E + \Delta C C_1^T\, X + Dw\big) - K_I C_2^T E + C_1 B a_u(t)\\ &= (A_1 - K_P C C_1^T - K_I C_2^T)E + \Delta A_1 X - K_P\,\Delta C\, C_1^T\, X + C_1 B a_u(t) + \big(C_2 - K_P\big)D\,w\end{aligned} \tag{7}$$

In the following steps, we will introduce a method to design K_P and K_I which guarantees the bounded stability of estimation error dynamics. This method considers four assumptions and is based on one theorem, which are coming in the next paragraphs.

Assumption 1. Magnitude of disturbance signal w is bounded

$$\|w\| \le w_1 \tag{8}$$

Where w_1 is a positive scalar.

Assumption 2. $\Delta A_1.X$ is bounded and $\Delta A_1.X \le \beta$. Here $\beta \in R^{(n+q)\times 1}$ is a known vector of this bound.

Assumption 3. $\Delta C C_1^T X$ is bounded and $\Delta C C_1^T X \le \gamma$ which $\gamma \in R^{q\times 1}$ is a known vector of this bound.

Assumption 4. $C_1 B u_a(t)$ is bounded and $C_1 B a_u(t) \le \alpha$ which $\alpha \in R^{(n+q)\times 1}$ is a known vector of this bound.

Theorem 1. Consider that assumptions 1 to 4 hold. If there exist positive definite matrices P , Q and positive scalars δ, σ , ρ, μ , matrices K_P and K_I such that the following equality holds:

$$\begin{aligned} &P(A_1 - K_P C C_1^T - K_I C_2^T) + (A_1 - K_P C C_1^T - K_I C_2^T)P \\ &\quad + \rho^{-1} P(C_2 D - K_P D)(C_2 D - K_P D)^T P \\ &\quad + \delta^{-1} P \beta \beta^T P + \sigma^{-1}(-PK_P\gamma)(-PK_P\gamma)^T \\ &+ \mu^{-1} P \alpha \alpha^T P = -Q \end{aligned} \tag{9}$$

Then the estimation error dynamics of equation (7) is uniformly ultimately bounded stable and the bound of estimation error is,

$$\|E(t)\| \le \left(\frac{\rho w_1^{\,2} + \delta + \sigma + \mu}{\lambda_m(Q)} \right)^{1/2} as\ t \to \infty \tag{10}$$

Where $\lambda_m(Q)$ represents the minimum eigenvalue of positive definite matrix Q. To prove theorem (1), the following lemmas have to be introduced:

Lemma 1. [22] Let H, F and G be real matrices of appropriate dimensions, then for any scalar $\varepsilon > 0$ and the matrix F satisfying $F^T F \le I$, we have

$$H F G + G^T F^T H^T \le \varepsilon H H^T + \varepsilon^{-1} G^T G \tag{11}$$

Lemma 2. [23] The Linear Matrix Inequality $\begin{bmatrix} H & S^T \\ S & R \end{bmatrix} > 0$ is equivalent to $R > 0$ and $H - S^T R^{-1} S > 0$ where $H = H^T$, $R = R^T$ and S is a matrix with appropriate dimension.

Proof of Theorem 1.

Define the following Lyapunov function

$$V = E^T PE \tag{12}$$

Such that P is a symmetric positive definite matrix ($P = P^T$, $P > 0$). Using error dynamics equation (6), the first derivative of V with respect to time can be obtained as:

$$\begin{aligned}
\dot{V} &= E^T P\dot{E} + \dot{E}^T PE \\
\dot{V} &= E^T [P(A_1 - K_P CC_1^T - K_I C_2^T) + (A_1 - K_P CC_1^T \\
&\quad - K_I C_2^T)P]E + 2E^T P\Delta A_1 X - 2E^T PK_P \Delta C\, C_1^T X \\
&\quad + 2E^T PC_1 Ba_u(t) + 2E^T P(C_2 - K_P)Dw
\end{aligned} \tag{13}$$

Using Lemma 1, the last term of this equation satisfied the following inequality:

$$\begin{aligned}
2E^T P(C_2 - K_P)Dw &= w^T I(C_2 D - K_P D)^T PE \\
&\quad + E^T P(C_2 D - K_P D) I w \\
&\le \rho^{-1} E^T P(C_2 D - K_P D)(C_2 D - K_P D)^T PE \\
&\quad + \rho w^T w
\end{aligned} \tag{14}$$

From Lemma 1 and assumption (2) we may conclude that:

$$\begin{aligned}
2E^T P\Delta A_1 X &= I(\Delta A_1 X)^T PE + E^T P(\Delta A_1 X) I \\
&\le \delta^{-1}(E^T P\Delta A_1 X)((\Delta A_1 X)^T PE) \\
&\quad + \delta(I I^T) \\
&\le \delta^{-1}(E^T P\beta)(\beta^T PE) + \delta I
\end{aligned} \tag{15}$$

Also, from Lemma 1 and assumption (3) we reach to the following inequality:

$$\begin{aligned}
-2EPK_P \Delta C\, C_1^T X &= I\left(-(\Delta C\, C_1^T X)^T K_P^T PE\right) + \left(-E^T PK_P \Delta C\, C_1^T X\right) I \\
&\le \sigma^{-1} E^T \left(-PK_P \Delta C\, C_1^T X\right)\left(-(\Delta C\, C_1^T X)^T K_P^T P\right) E \\
&\quad + \sigma(I.I^T) \\
&\le \sigma^{-1} E^T (-PK_P \gamma)(-\gamma^T K_P^T P) E + \sigma I
\end{aligned} \tag{16}$$

At last, from Lemma 1 and assumption (4) we reach to

$$
\begin{aligned}
2E^T PC_1 Ba_u(t) &= I\left(C_1 Ba_u(t)\right)^T PE + E^T P\left(C_1 Ba_u(t)\right) I \\
&\le \mu^{-1}(E^T PC_1 Ba_u(t))(\left(C_1 Ba_u(t)\right)^T PE) \\
&\quad + \mu(I\, I^T) \\
&\le \mu^{-1}(E^T P\alpha)(\varepsilon^T PE) + \mu I
\end{aligned} \tag{17}
$$

Substituting inequalities (14) to (17) in equation (13) and using equation (9), it can be easily concluded that ,

$$
\begin{aligned}
\dot{V} &\le -E^T QE + \rho w^T w + \delta + \sigma + \mu \\
&\le -\|E\|^2 \lambda_m(Q) + \rho w^T w + \delta + \sigma + \mu
\end{aligned} \tag{18}
$$

a. If the right side of inequality (18) is less than zero:

$$
\dot{V} \le -\|E\|^2 \lambda_m(Q) + \rho w^T w + \delta + \sigma + \mu < 0 \tag{19}
$$

Then

$$
\|E(t)\| > \left(\frac{\rho w_1{}^2 + \delta + \sigma + \mu}{\lambda_m(Q)} \right)^{1/2} \tag{20}
$$

This shows that

$$
\|E(t)\| = \left(\frac{\rho w_1{}^2 + \delta + \sigma + \mu}{\lambda_m(Q)} \right)^{1/2} \tag{21}
$$

define a hyper sphere which out of that (showed by equation (21), stability condition $\dot{V} < 0$ holds.

b. If the right side of inequality (19) is equal to zero, then on the hyper sphere of equation (21) $\dot{V} \le 0$ holds. $\dot{V} = 0$ Shows the existence of a limit cycle [24-25] and $\dot{V} < 0$ shows the stability of estimation error dynamics.

c. If the right side of inequality (19) is greater than zero, then

$$
\|E(t)\| < \left(\frac{\rho w_1{}^2 + \delta + \sigma + \mu}{\lambda_m(Q)} \right)^{1/2} \tag{22}
$$

It shows that inside the hyper sphere of equation (21), $\dot{V}$ is less than or equal to a positive value. Therefore further comments could not be given on the stability of estimation error dynamics inside the mentioned border.

From three situations of a, b and c we conclude that equation (21) is an attractor for estimation error dynamics (7) and the system is at least uniformly ultimately bounded stable.

Equation (9) can be rewritten in LMI notation as follows:

$$\begin{bmatrix} G & P\beta & -P(C_2D-K_PD) & -PK_P\gamma & -P\alpha \\ \beta^TP & -\delta & 0 & 0 & 0 \\ -(C_2D-K_PD)^TP & 0 & -\rho & 0 & 0 \\ -(PK_P\gamma)^T & 0 & 0 & -\sigma & 0 \\ -(P\alpha)^T & 0 & 0 & 0 & -\mu \end{bmatrix} < 0$$

$$\delta > 0$$
$$\rho > 0$$
$$\sigma > 0$$
$$\mu > 0$$
$$P = P^T > 0 \qquad (23)$$

Where

$$G = P(A_1 - K_PCC_1^T - K_IC_2^T) + (A_1 - K_PCC_1^T - K_IC_2^T)P \qquad (24)$$

Using LMI tool box in MATLAB, we can easily find the solution of $P,\ K_P,\ K_I,\ \delta,\ \rho, \mu,\ and\ \sigma$.

4 State Estimation Design Procedure

The overall design procedure can be summarized as follows

Data: Uncertain system including actuator attack (Eq. 1)

Goal: Design an observer based on inputs of the plant and output measurements.

I. Obtain A_1, C_1 and C_2 in equation (3).
II. Determine the upper bounds of $\Delta A_1 X, C_1Ba_u(t)$ and $\Delta C C_1^T X$ such that $\Delta A_1.X \le \beta$, $C_1Ba_u(t) \le \alpha$ and $\Delta C C_1^T X \le \gamma$ in assumptions 2 and 3.
III. Construct LMI rules (24) with variables $\delta, \rho, \sigma, \mu, P, H_1 = PK_P$ and $H_2 = PK_I$.
IV. Using LMI tool box in MATLAB, find K_P and K_I .
V. Calculate $K_P = P^{-1}H_1$ and $K_I = P^{-1}H_2$
VI. Construct the PI-Observer of equation (8).

Remark 1. For some systems which $\Delta A(t).x(t) = f(x,t)$, $\Delta C(t).x(t) = g(x,t)$ and two funtion $f(x,t)$ and $g(x,t)$ are bounded the proposed method can be used for designing a stable and error-free observer in nonlinear systems of following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x,t) + B\big(u(t) + a_u(t)\big) \\ y(t) &= Cx(t) + g(x,t) + Dw \end{aligned} \qquad (25)$$

Remark 2. Luenberger observer for system in matrix form (Eq. 3) can be written as following:

$$\dot{\hat{X}} = A_1\hat{X} + C_1Bu + K_P(Y_1 - CC_1^T\hat{X}) \qquad (26)$$

the equation of error dynamics is obtained as:

$$\dot{E} = (A_1 - K_PCC_1^T)E + \Delta A_1X - K_P\Delta CC_1^TX + (C_2 - K_P)Dw + C_1Ba_u(t)$$

(27)

It can be seen that if the desired eigenvalue of $(A_1 - K_P C C_1^T)$ are chosen in the neighberhood of the orgin, the integral behavior of the error dynamics is increased, but it degrade the performance of the observer greatly. On the other hand, if the eignevalues are set far from the origin, it increases the system performance but it increases the unwanted effects of model mismatch term, diturbamce term, and attack term, too.

Remark 3. As is seen in equation (7), due to the presence of model mismatches ΔA and ΔC , disturbance w, and false data injection $a_u(t)$, the state estimation error does not always converge to zero.

Remark 4. For presented system in (1), the term $Ba_u(t)$ is false data injection (FDI) attack on input or diturbed input. Furthermore, the term Dw can be behaviored as false data injection on output. The proposed algorithm can estimate the states of system accurately if the assumption.1 is valid.

5 Numerical Example

To show the ease of implementation and robust performance of the introduced method, a numerical example is studied in this section. It should be noted that the present example is a physical example to verify the robustness of the presented algorithms regardless of its application in cyber-physical systems since the proposed approach can be implemented for a different range of systems. Considering a nonlinear pneumatic artificial muscle (PAM) presented in [26]. The system is a nonlinear system and the PAM force depends on the air pressure, dimensions, and properties of the muscle materials. It is assumed that the input signal would be disturbed as false data injection by two different signals: random and sinusoidal. It should be noted that the presented model has a model mismatch; however, a model mismatch is assumed on output. The schematic of the system is shown in figure 1. The equation of motion of the system by non-dimensional parameters can be written as follows.

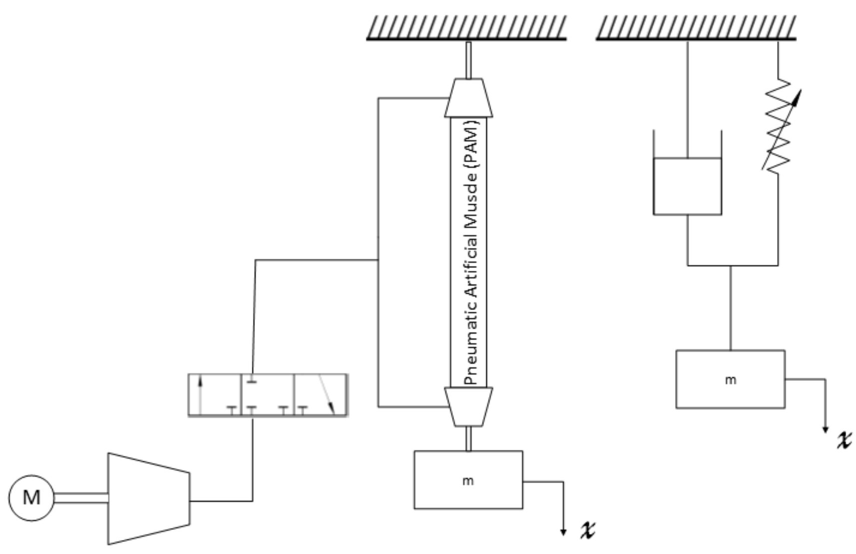

Figure 1- the schematic model of pneumatic artificial muscle

$$\ddot{x} + 2\varepsilon\varphi\dot{x} + x + \varepsilon(p_1 \sin\Omega\tau + p_2 \cos 2\Omega\tau\,)x + \varepsilon\theta x^3 = \varepsilon(f_1 + f_2 \sin\Omega\tau) \tag{28}$$

Where;

$$P = P_m + P_0 sin\bar{\Omega}\tau, \qquad \omega_0 = \sqrt{\frac{\left[c_1 + c_2 P_m + c_3 (P_m^2 + \frac{P_0^2}{2})\right]}{m l_{max}}}, \qquad \Omega = \frac{\bar{\Omega}}{\omega_0},$$

$$\varphi = \frac{c}{2\varepsilon m \omega_0}, \qquad \theta = \frac{r^2 K}{\varepsilon m \omega_0^2}, \qquad p_1 = \frac{c_2 P_0 + 2c_3 P_0 P_m}{\varepsilon m \omega_0^2 l_{max}}, \qquad p_2 = -\frac{c_3 P_0^2}{2\varepsilon m \omega_0^2 l_{max}}$$

$$f_1 = \frac{d_2 P_m + d_1 - mg}{\varepsilon m r \omega_0^2}, \qquad f_2 = \frac{d_2 P_0}{\varepsilon m r \omega_0^2}$$

The defined parameters and their values are listed in Table 1.

Table 1-List of parameters

No.	Parameter	Description	Units	Value
1	m	Mass of load on muscle	kg	6
2	l_{max}	Maximum length of muscle	m	0.074
3	c_1	Force-Displacement Constant	N	-234.25
4	c_2	Force-Displacement Constant	N/pa	0.00196
5	c_3	Force-Displacement Constant	N/pa^2	-3.00E-09
6	d_1	Force-Displacement Constant	N	-100
7	d_2	Force-Displacement Constant	N/pa	0.001
8	K	Force-Displacement Constant	N/m^3	16490
9	c	Damping coefficient	N.s/m	0.08
10	$\bar{\Omega}$	Frenquency of input pressure	1/s	13.509
11	P_m	Mean value of input pressure	pa	300000
12	P_0	Amplitude of input pressure	pa	40000
13	r	Scaling factor	1/m	1
14	ε	Non-dimensional Parameter	-	0.1
15	ω_0	Fundamental natural frequency of PAM	1/s	13.536
16	Ω	Frequency ratio	-	0.998
17	φ	Non-dimensional Parameter	-	0.00493
18	θ	Non-dimensional Parameter	-	150
19	p_1	Non-dimensional Parameter	-	0.787
20	p_2	Non-dimensional Parameter	-	0.295
21	f_1	Non-dimensional Parameter	-	1.2844
22	f_2	Non-dimensional Parameter	-	0.364

According to presentend nonlinear state space, the system can be transformed to its form of equation (1), then the system matrice can be easily obtained as:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2\varepsilon\varphi \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0]$$

Here we assumed that the nonlinear part of PAM is a model mismatch, so $\Delta A.X = -\varepsilon(p_1 sin\Omega\tau + p_2 cos2\Omega\tau)z_1 - \varepsilon\alpha z_1^3$. Refer to equation (5), matrices A_1, C_1 and C_2 can be obtained as:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2\varepsilon\varphi & 0 \\ 1 & 0 & 0 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

In this system, we can assume that $\Delta A_1.X$ is a nonlinear term which is a bounded term, $\Delta A_1.X \leq [2.123, 2.123, 2.123]^T$. The efficiency of the proposed PI-observer is evaluated for different conditions, as follows:

Case I: $a_u = 0, \Delta C = 0, D = 0$

The assumed conditions means the simulation of presented model in [24] without output mismatch ($\Delta C = 0$), actuator attack ($a_u = 0$), and disturbance ($D = 0$). The gain matrices of PI-observer K_P and K_I may easily obtain the following results using the MATLAB-LMI toolbox:

$$K_P = \begin{bmatrix} 3.2103 \\ 2.4233 \\ 2.5640 \end{bmatrix} \text{ and } K_I = \begin{bmatrix} 13.1451 \\ 12.0252 \\ 12.5935 \end{bmatrix}$$

Figure 2 and 3 show the states of the system and their estimation. The figure shows that the estimation error quickly converge to zero.

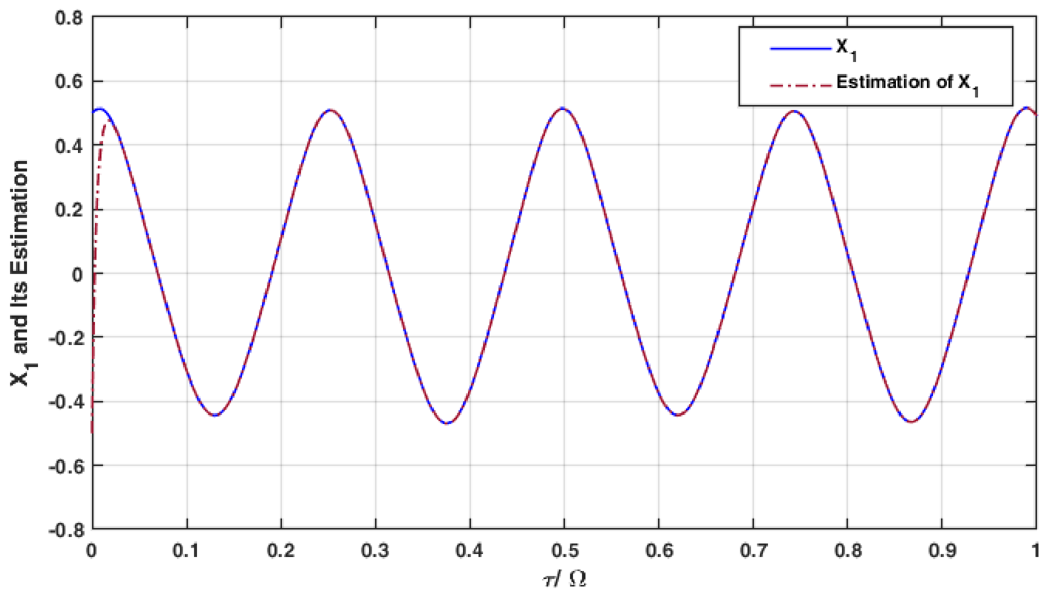

Figure 2- X_1 and its Estimation

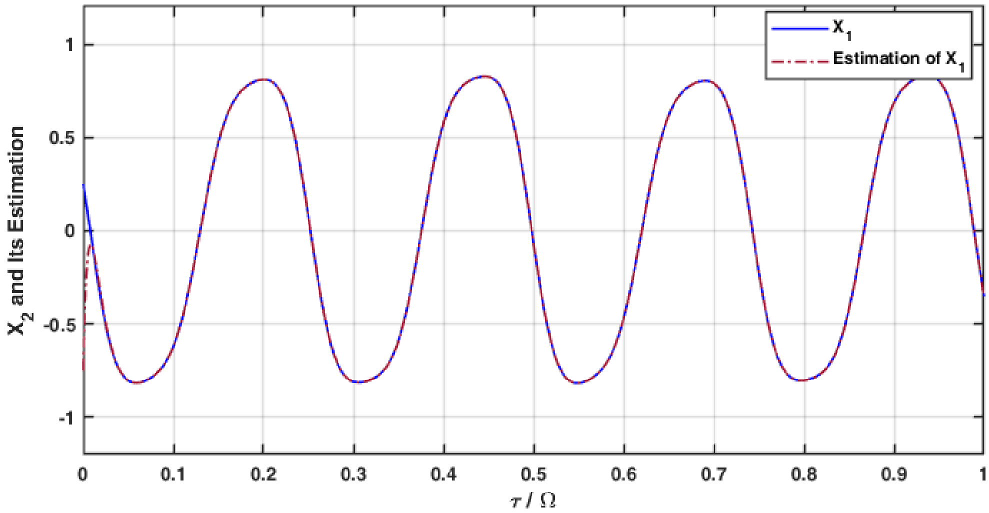

Figure 3- X_2 and its Estimation

Case II: $a_u = 0, \Delta C \neq 0, D \neq 0$

Suppose that there are some model mismatches on output as, $\Delta C = [0.05\sin(\tau) \quad 0.05\cos(\tau)]$, and a sinusoidal disturbance signal as $w = \cos(\pi t)$ with $D = 0.4$. In this condition we can obtain that that $\Delta C C_1^T X \leq 0.0673$. The gain matrices of PI-observer K_P and K_I may obtain the following results:

$$K_P = \begin{bmatrix} 3.7648 \\ 2.9889 \\ 3.1747 \end{bmatrix} \text{ and } K_I = \begin{bmatrix} 12.5184 \\ 11.4417 \\ 12.0307 \end{bmatrix}$$

Figure 4 and 5, illustrates the states of the system and their estimation. According to the figure, the error dose not converge to zero (**Remark 3**), however the average, maximum and minimum value of error are 0.0021, 0.00504, and 0.0546 respectively and$|x(t) - \hat{x}(t)| \leq \epsilon$, that means the error signal is bonded.

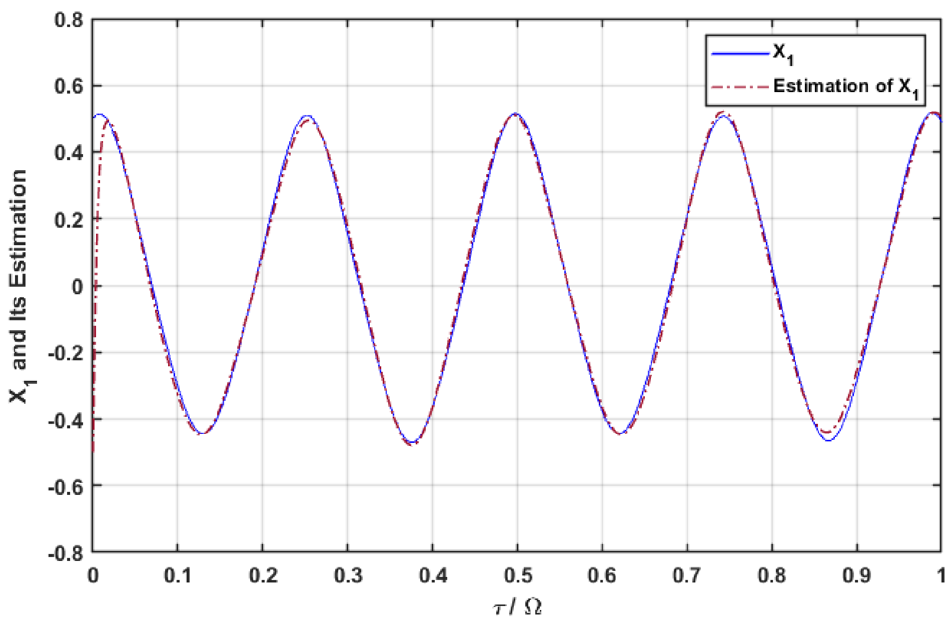

Figure 4- X_1 and its Estimation

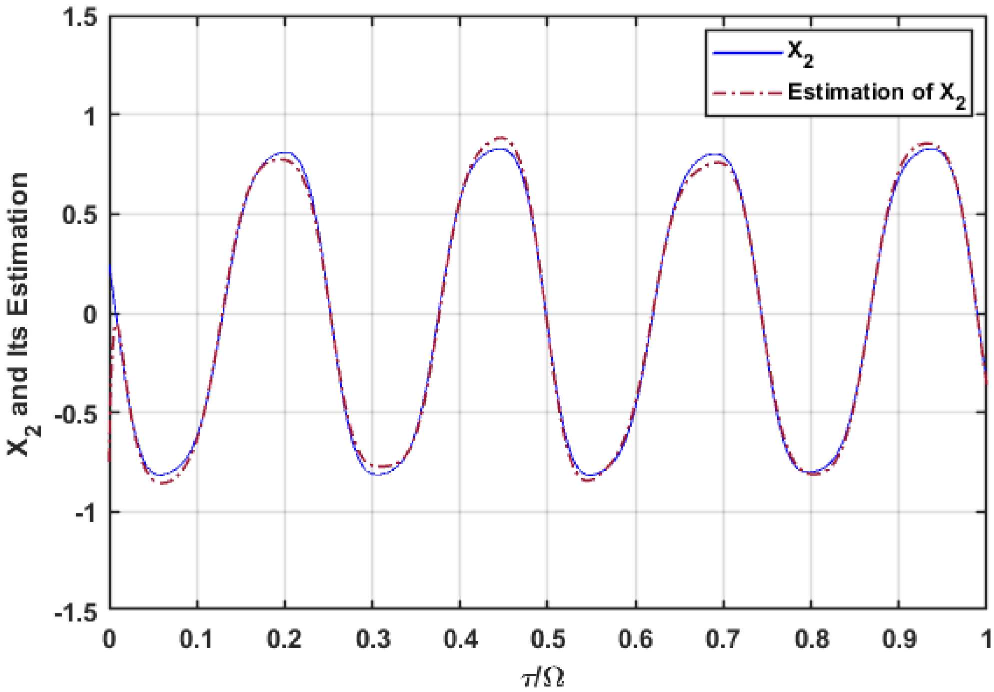

Figure 5- X_2 and its Estimation

Case III: $a_u \neq 0, \Delta C \neq 0, D \neq 0$

Suppose that an attack signal is added to Case II on input as, $a_u = 0.1\sin(\pi\tau)$. In this situation, we can obtain that $C_1 B a_u(t) \ \leq 0.1648$. The gain matrices of PI-observer K_P and K_I may obtain the following results:

$$K_P = \begin{bmatrix} 3.783 \\ 3.0063 \\ 3.193 \end{bmatrix} \text{ and } K_I = \begin{bmatrix} 12.5868 \\ 11.5101 \\ 12.0991 \end{bmatrix}$$

Figure 5 and 6, illustrates the states of the system and their estimation. According to the figure, the error dose not converge to zero (**Remark 3**); however the average, maximum and minimum value of error are 0.266, 0.00467, and 0.5276 respectively and$|x(t) - \hat{x}(t)| \leq \epsilon$, that means the error signal is bonded. As seen, although the attack on input signal increases the error of estimation, it is bonded and has acceptable range regarding the assumptions of proposed algorithm.

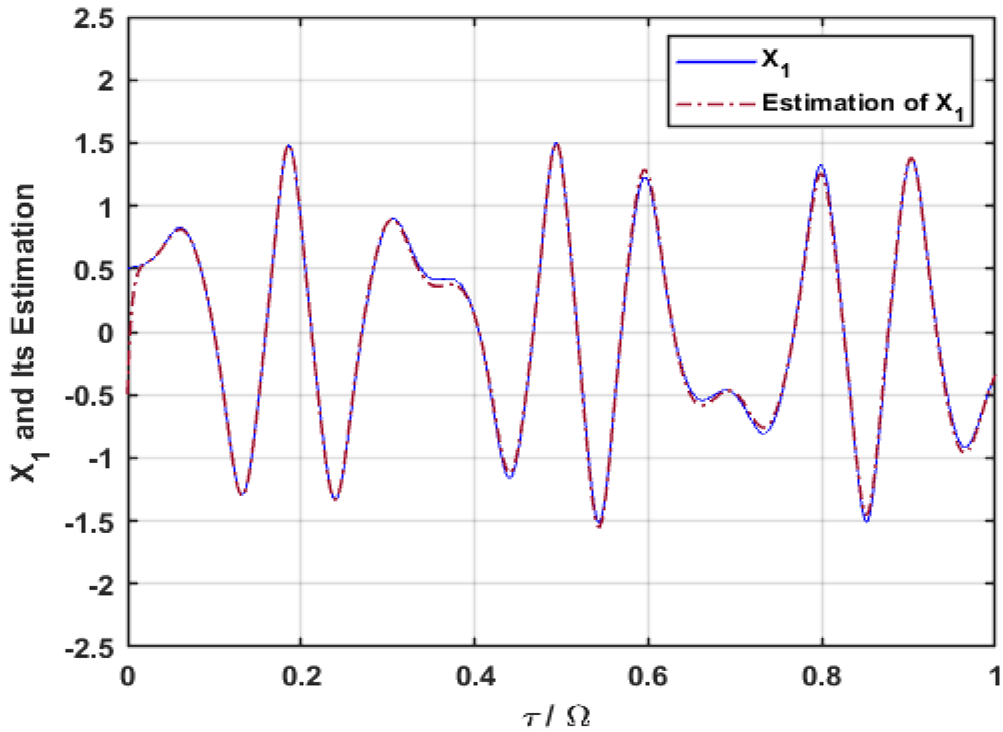

Figure 6- X_1 and its Estimation

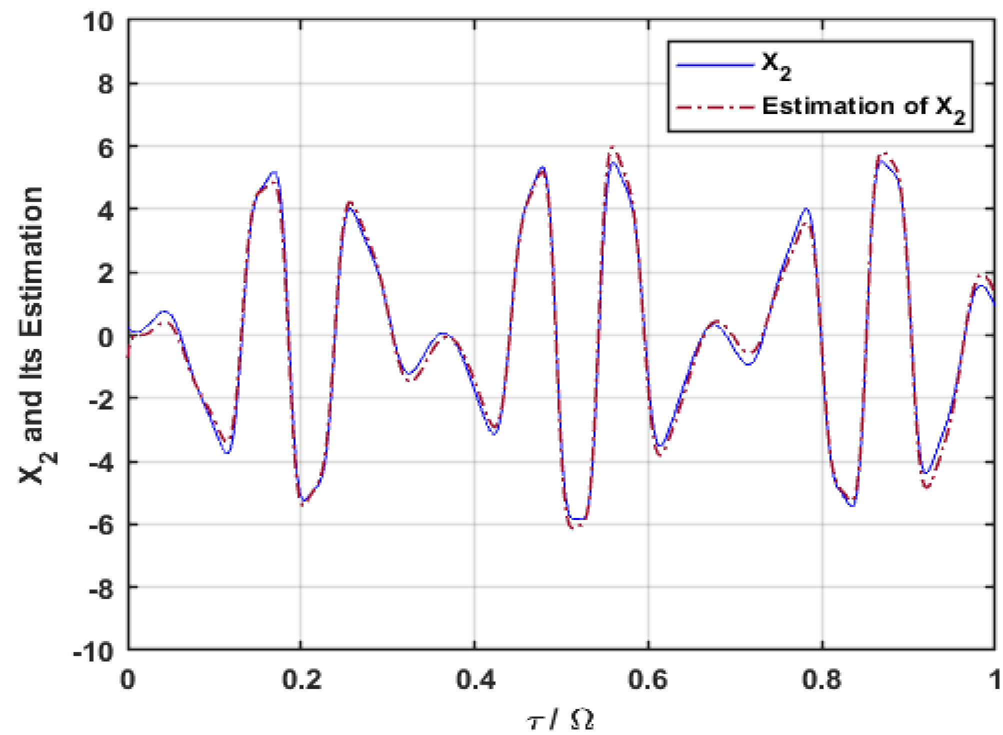

Figure 7- X_2 and its Estimation

Case IV: random, $w(t)$, $u_a(t)$ and $\Delta C \neq 0$

Suppose that the attack and disturbance signals are generated randomly according to Gaussian distributions with zero mean and covariance 10. The gain matrices of PI-observer K_P and K_I may obtain the following results:

$$K_P = \begin{bmatrix} 3.7725 \\ 3.1231 \\ 3.2911 \end{bmatrix} \text{ and } K_I = \begin{bmatrix} 12.476 \\ 11.6132 \\ 12.1712 \end{bmatrix}$$

Figure 7 and 8, illustrates the states of the system and their estimation. According to the figures, the error dose not converge to zero (**Remark 3**), however the average, maximum and minimum value of error are 0.0474, 0.0768, and 0.0181 respectively for 50 experiments and $|x(t) - \hat{x}(t)| \leq \epsilon$, that means the error signal is bonded. As seen, although the attack on input signal increases the error of estimation, it is bonded and has acceptable range regarding the assumptions of proposed algorithm.

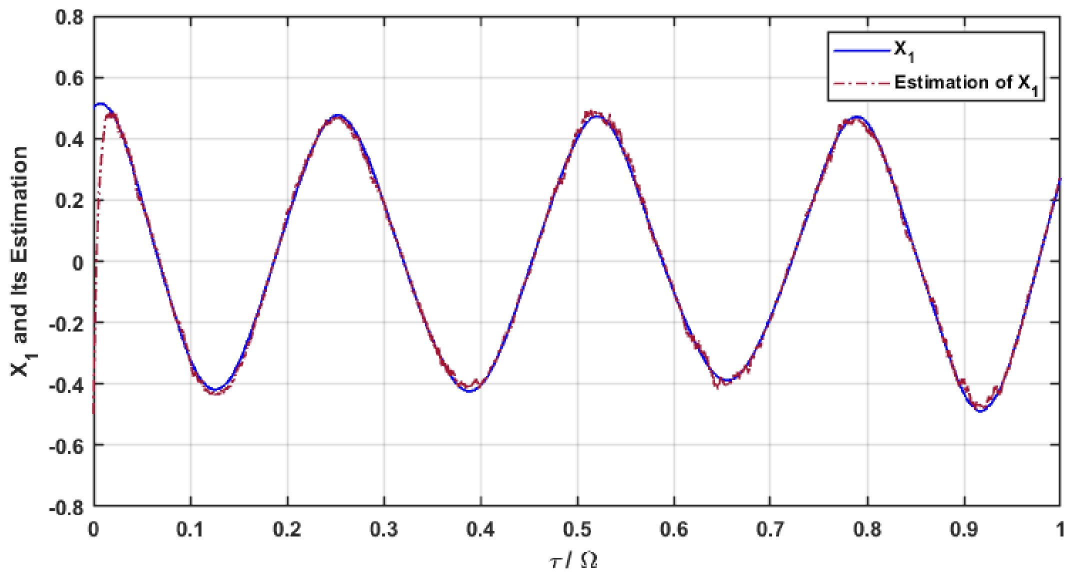

Figure 8- X_1 and its Estimation

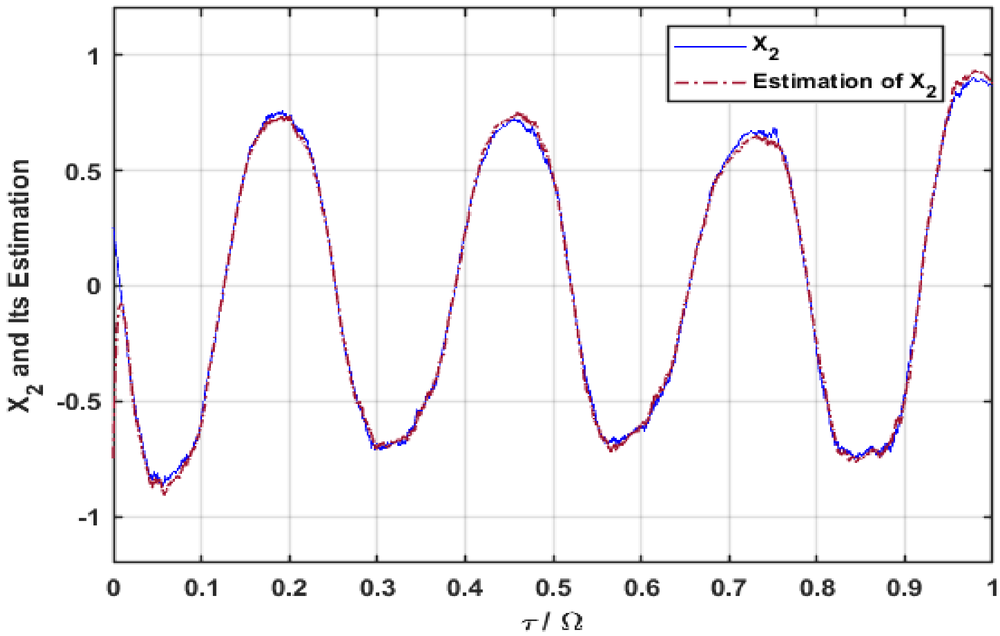

Figure 9- X_2 and its Estimation

6 Conclusion

In the present study, a general design algorithm for PI observers has been proposed for a MIMO LTI system under disturbance, model mismatch, and false data injection on the input signal. First, it is necessary to show the stability of the method. Thus, the Lyapunov theory using LMI approach is used to prove the stability of the proposed method. This design method guarantees the stability of the observer when the energy of disturbance signal and upper bound of model mismatch and actuator attack are limited. Then, the procedure of the observer designed is introduced to obtain the gain of the observer. Finally, a nonlinear pneumatic artificial muscle example has been given to illustrate the effectiveness of the observer under different conditions and types of false data injection. For future works, the proposed method can be extended for tolerant control, attack on the measurement, and model mismatch on input.

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