EasyChair Preprint

№ 5291

Lot Sizing Decisions Under Uncertain Demand Considering Skewness and Kurtosis

Fernando Rojas

April 6, 2021

Lot sizing decisions under uncertain demand considering skewness and kurtosis

Abstract

The lot sizing model is useful for supply making decisions based on probabilistic modeling of demand, using two-stage stochastic programming, calculating the optimal costs of a supply model. In this paper, we study this model by using simulated scenarios subject to different degrees of skewness and kurtosis to model demand, considering univariate Weibull statistical distribution described by a generalized additive models of location, scale and shape (GAMLSS).

We carried out a simulation study of 10,000 different demand scenarios with different degrees of skewness and kurtosis, evaluating relationships between total costs, lot size decisions, expected stock and out of stock respect to coefficients of demand skewness and kurtosis.

In this study it has been shown that the coefficients of skewness and kurtosis impact on the total costs of supplying an item. The results also allow generating a predictive pattern of the first and second stage decisions, that is, the expected quantities in stock and shortages for the use of stochastic lot sizing. Our results indicate that the higher total cost of supply and greater shortage are related to demand patterns with more negative symmetry and lower kurtosis.

Keywords: GAMLSS; kurtosis; lot sizing; statistical moments, skewness, Weibull statistical distribution.

1 Introduction

An inventory management system is essential to give structure and direction to the decision-making of an organization regarding the supply, see Sabet et al. (2020). In inventory management, the costs of purchasing, ordering, holding and shortages are determined, requiring an estimate of demands in a certain time whose restriction can be of different types: budgetary, by storage volume, service levels, among others Rojas et al. (2019).

Stochastic programming is an approximation to model optimization problems that involve uncertainty (Shapiro et al., 2014). While deterministic optimization problems are formulated with known parameters, real-world problems almost invariably include parameters that are unknown at the time decisions must be made. When the parameters are uncertain, but they are assumed to be within a set of possible values, a solution could be sought that is feasible for all possible choices of parameters and optimizes a given objective function (Raa and Aghezzaf, 2005). In this way, it is possible to have extensions of lot sizing models in probabilistic environments, through an approach based on two-stage stochastic programming (SP), obtaining first-stage decisions without yet knowing the realization of a random variable, such as demand per unit of weather. Subsequently, in a second stage, stock decisions and probable shortages are obtained, considering the generation of stochasticity scenarios for the demand of a product (Rojas et al., 2019).

To the best of our knowledge, the lot sizing models has been mostly studied assuming independent and normally distributed demands per unit time (DPUT) of the items, but DPUTs are random variables (RVs) that may show any shape (Sadeghi and Niaki, 2015). For the statistical distributions of RV independently and identically distributed (IID), such as the DPUTs, a statistical moment is a particular calculable dimension of the shape of its probability density functions (PDFs). The zero-th moment is always 1, the 1-th moment is the mean, the 2-th central moment is the variance, the 3-th standardized moment is the skewness, while the 4-th standardized moment is the kurtosis; see Casella and Berger (2002), Deng et al. (2020) and Lin et al. (2020). As we will see later in the background of this paper, all these moments are related and for this paper we will postulate that they can influence inventory lot sizing.

Often an RV data set has a joint statistical distribution based on marginal distributions with different skewness and kurtosis, as noted in Escribano and Pfann (1998) and Chan et al. (2005). In this context, it is essential to have excellent goodness of fit to actual data for theoretical description of the marginal statistical distributions (Alavifard, 2019; Zhi et al., 2020).

Generalized Additive Model for Location, Scale and Shape (GAMLSS) is a semi-parametric regression type model introduced by Stasinopoulos and Rigby (2007) that allows great versatility in the modeling of RVs (Rohmer and Gehl, 2020).

The main objective of this paper is to propose a new methodology for studying how the skewness and kurtosis of marginal distributions of DPUTs affect the total costs (TC) and inventory decisions under a stochastic lot sizing model in two-stage. The remainder of the paper is organized as follows. Section 2 presents the methodology proposed in this study built upon two pillars: (i) modelling of marginal statistical distributions of DPUTs with different skewness and kurtosis; and (ii) stochastic lot sizing model in two-stage. A simulation study is performed in Section 3 to analyze how the marginal statistical distribution of DPUTs with different skewness and kurtosis affect the TC and inventory decisions of stochastic lot sizing. In Section 4 finishing with a discussion and the conclusions of the results obtained in this research, along with their limitations and possible future research.

2 Methodology

As mentioned in the introduction, it is possible to use the GAMLSS model to model the marginal distributions of random variables. Unlike generalized linear models (GLM), the GAMLSS considers a family of generalized, discrete, or continuous statistical distributions, which can have varying degrees of skewness and kurtosis. Thanks to its formulation, it is possible to model any parameter of these statistical distributions for the response variable linearly or not in an additive parametric or non-parametric form of covariates with known or random values; see Stasinopoulos et al. (2008). The advantage of using this type of statistical modeling is that GAMLSS is a regression toolbox appropriate for a big dataset of response variables that can consider linear or smoothing functions of predictive covariates to model any parameter of location, scale, or shape of the statistical distribution. The current packages available in R software (Stasinopoulos et al., 2015), allow working with continuous (any type of skewness or kurtosis), discrete (including zero inflated data), and mixture statistical distributions. Models can be selected according to criteria of goodness of fit to the real data, as well as by generating random numbers with arbitrary distributions of interest for theoretical or empirical research (Rojas et al., 2019; Rojas and Ibacache-Quiroga, 2020; Rojas et al., 2020). Our proposal differs from what has been addressed in the literature on the stochastic lot sizing of inventories. We describe IID DPUT as a marginal statistical distribution described by GAMLSS models, making it possible to generate different skewness and kurtosis to explore the TC and two-stage decisions that leads a stochastic lot sizing.

The new methodology proposed in this paper is subdivided into the following two sections:

- Section 2.1 shows how to describe the DPUT for an inventory item using GAMLSS, moments, skewness and kurtosis and their use with Weibull type 3 statistical distribution.
- Section 2.2 provides the elements of a probabilistic lot size inventory model using SP in two-stages.

2.1 How to describe the DPUT for an inventory item

GAMLSS formulation Let Y be an IID RV corresponding to the DPUT. If Y be the DPUT of an inventory item, we considered that μ is the expected value of a response variable. Consider to d as a covariate. If $f(y|\boldsymbol{\theta})$ be a conditional PDF on parameters $\boldsymbol{\theta}$ ($F_{y|\boldsymbol{\theta}}$ is the conditional cumulative distribution function (CDF)), where $\boldsymbol{\theta} = (\mu, \sigma, \nu, \tau)^\top = (\theta_1, \theta_2, \theta_3, \theta_4)^\top$ is a vector of four distribution parameters. In the GAMLSS formulation, only μ is a function of the covariates, μ and σ are location and scale parameters, and ν and τ are shape parameters. If $\{y_i\}, i = 1, \ldots, n$ is an $n \times 1$ vector of the response variable to model, considering $k = 1, 2, 3, 4$ as parameters. Then g_k is a link functions related to the k-th parameter $\boldsymbol{\theta}_k$ to covariates by additive models:

$$
\begin{aligned}
g_1(\boldsymbol{\mu}) &= \boldsymbol{\eta}_1 = \boldsymbol{D}_1\boldsymbol{\beta}_1 + \sum_{j=1}^{J_1} h_{j1}(\boldsymbol{d}_{j1}), && (1)\\
g_2(\boldsymbol{\sigma}) &= \boldsymbol{\eta}_2 = \sum_{j=1}^{J_2} h_{j2}(\boldsymbol{d}_{j2}), && (2)\\
g_3(\boldsymbol{\nu}) &= \boldsymbol{\eta}_3 = \sum_{j=1}^{J_3} h_{j3}(\boldsymbol{d}_{j3}), \text{ and} && (3)\\
g_4(\boldsymbol{\tau}) &= \boldsymbol{\eta}_4 = \sum_{j=1}^{J_4} h_{j4}(\boldsymbol{d}_{j4}), && (4)
\end{aligned}
$$

where $\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\nu}, \boldsymbol{\tau}, \boldsymbol{\eta}_t$ and $\boldsymbol{d}_{j1}$, for $j = 1, \ldots, J_k$ and $k = 1, 2, 3, 4$, are $n \times 1$ vectors. $\boldsymbol{D}_1$ is an $n \times J_1$ known matrix of variables and the regression coefficients $\boldsymbol{\beta}_1$ to be estimated is a $J_1 \times 1$ vector. h_{jk} is a semi-parametric additive function for the covariate D_{jk} evaluated at the vector $\boldsymbol{d}_{jk}$, which is assumed fixed and known.

For details of parameter estimate, diagnostic and good fit on the data see Stasinopoulos and Rigby (2007)

Moments, Skewness, and Kurtosis If F is a CDF of any statistical distribution, considering the Riemann Stieltjes integral (Liu, 2004), the n-th moment of the statistical distribution is expressed by:

$$\mu'_n = \mathrm{E}\left[Y^n\right] = \int_{-\infty}^{\infty} y^n \, \mathrm{d}F(y)$$

with E as an expectation operator for the mean.

The zero-th moment of any PDF is 1.

The first raw moment is the mean:

$$\mu \equiv \mathrm{E}[Y].$$

The second central moment is the variance, and the square root of the variance is the SD:

$$SD \equiv \left(\mathrm{E}\left[(y-\mu)^2\right]\right)^{\frac{1}{2}}.$$

The normalized n-th central moment of the RV Y is

$$\frac{\mu_n}{SD^n} = \frac{\mathrm{E}\left[(Y-\mu)^n\right]}{SD^n},$$

and represents the distribution.

The normalized third central moment is called the skewness. A distribution that is skewed to the left has a negative skewness, and vice versa. Zero values indicate symmetry of the distribution. The Fisher coefficient of skewness (CSk), is defined as:

$$CSk = \frac{\mu'_3}{SD^3},$$

where μ'_3 is the third moment centered.

The fourth central moment is a measure of outliers values far from the average distribution values and is denominated kurtosis. Statistical distributions with kurtosis less than 3 are said to be " platykurtic ", while distributions with kurtosis greater than 3 are said to be " leptokurtic ". The Fisher coefficient of skewness (CK), is defined as:

$$CK = \frac{\mu'_4}{SD^4},$$

where μ'_4 is the fourth moment centered.

The parameterization of the probability density function (PDF) for Weibull type 3 statistical distribution (WEI3) in GAMLSS environment is given by

$f(y|\mu,\sigma) = (\sigma/\beta) * (y/\beta)^{(\sigma-1)} exp(-(y/\beta)^{\sigma})$

where $\beta = \mu/(\Gamma(1/\sigma)+1)$ for $y > 0, \mu > 0$ and $\sigma > 0$ are parameters of GAMLSS showed in 2.1 (Stasinopoulos et al., 2020). We can also calculate the variance (sd_Y^2) as:

$\mathrm{sd}_Y^2 = \mu_Y^2 \{ \frac{\Gamma(\frac{2}{\sigma_Y+1})}{\Gamma(\frac{1}{\sigma_Y+1})^2} - 1 \}.$

2.2 Probabilistic lot size inventory model using SP in two-stages

Parameters	Variables
t: Period index of the decision stage in the planning time horizon ($t = 1, \ldots, T$).	Z_t: Binary variable indicating whether a purchase is carried out in period t or not.
C_t: Purchase budget in period t.	Q_t: Quantity of units to be purchased in period t .
u_t: Unitary cost of purchase in period t.	I_t: Stock level at the end of period t.
o_t: Fixed order cost in period t.	I_0: Initial stock level.
h_t: Holding cost at the end of period t .	S_t: Shortage level at the end of period t.
s_t: Shortage cost at the end of period t.	
p_t^ω: Probability of occurrence of the scenario ω in period t of the decision stage.	

To obtain the observed values of a forecast DPUT y_t^ω of Y_t, their probabilities p_t^ω, and E(TC), we adapted method showed in Rojas et al. (2019) to GAMLSS environment. The corresponding SP framework used to minimize the expected TC -E(TC)- of the inventory model can be formulated as Raa and Aghezzaf (2005)

$$\min\{\mathrm{E(TC)}\} = \min\left\{\sum_{\omega\in\Omega}\sum_{t=T}^{T+1} o_t Z_t + u_t Q_t + p_t^\omega (h_t I_t^\omega + s_t S_t^\omega)\right\}, \tag{5}$$

subject to

$$\begin{aligned} Q_t + (I_{t-1}^\omega - S_{t-1}^\omega) - (I_t^\omega - S_t^\omega) &= y_t^\omega, \\ Q_t &\leq C_t Z_t, \end{aligned} \tag{6}$$

$$\forall t \in T,\ \ \forall \omega \in \Omega,\ \ Q_t \geq 0,\ \ I_t^\omega \geq 0,\ \ S_t^\omega \geq 0,\ y_t^\omega \geq 0,\ \ p_t^\omega \in [0,\,1],\ \ Z_t \in \{0,\,1\},$$

where Ω is the set of selected possible demand scenarios and ω is a specific scenario, with a fixed number of scenarios in each period of the decision stages. The objective function defined in Eq. (5) attains a solution that minimizes E(TC) over all scenarios. This minimization can be carried out through the addition of sharing cuts for feasibility and optimality at the resource function, whenever this function or its constraints contain stochastic coefficients in a multi-stage problem Infanger and Morton (1996).

3 Simulation and analysis of results

The new methodology proposed in this paper is studied using simulation and corroborated by an illustrative actual case. Here, we consider the following three sections:

- Section 3.1 presents details of the computational framework utilized and describes the simulation scenarios, which is divided into two parts as indicated below.

- Section 3.2 provide the results of simulation study where we analyzed the performance of a new methodology of probabilistic lot sizing on inventory with different skewness and kurtosis for the demand.

3.1 Computational framework and simulation scenarios

We implemented our proposal in a non-commercial software named R; see http://www.r-project.org. See (Rojas et al., 2015, 2020; Wanke and Leiva, 2015; Wanke et al., 2016) to visualize R applications in supply models .

The simulation of 10.000 scenarios establish different: (i) inventory policies, (ii) statistical models for the DPUT, and (iii) SP to minimize costs. First, we used probabilistic lot sizing model (Rojas et al., 2019). Second, the statistical modeling is based on IID DPUTs assuming a Weibull type 3 (WEI3) statistical distribution, see Stasinopoulos et al. (2020). We assumed several structures of skewness and kurtosis with WEI3 marginal statistical distributions for the DPUT. The uniformly distributed parameters employed to build these scenarios are chosen from values found in selected papers; see Table 2 and Appendix C of Wanke (2014) for a list of these values. Third, SP is performed to minimize the total cost of inventory by using an objective function showed in subsection 2.2.

We used the following indicators with WEI3 statistical distribution to generate these 10,000 scenarios and inventory policies obtained by SP in two-stage:

Statistical parameters

- $\mu_Y \sim \mathrm{U}(300, 1200)$,
- $\sigma_Y \sim \mathrm{U}(10, 60)$,

Inventory parameter

- $h_t \sim \mathrm{U}(0.25, 0.33)$,
- $o_t \sim \mathrm{U}(12000, 18000)$,
- $u_t \sim \mathrm{U}(15000, 30000)$,
- $s_t \sim \mathrm{U}(700, 2000)$,
- $C_t \sim \mathrm{U}(10000000, 20000000)$,

The choice of these values is based on previous studies on the topic; see Wanke (2009), Wanke and Saliby (2009) and Wanke (2014).

3.2 Simulation Study

Firstly, in the 10,000 scenarios proposed their skewness and kurtosis frameworks for the DPUTs, we analyzed to respect that probabilistic lot sizing that provide the minimum total cost, and their decisions in first and second stage. Table 1 show descriptive statistical indicators of 10000 scenarios of the simulation study

Table 1: Descriptive statistical indicators of 10000 scenarios of the simulation study

Indicator	μ_Y	σ_Y	sd	CSk	CK	Q	I	S	TC	o	h
Min.	300.1	10.01	688	-1.7274	2.926	280.5	3.114	2.499	4791629	12003	0.2500
1st Qu.	454.3	22.47	1764	-1.0365	3.958	441.4	10.106	9.899	40749989	13481	0.2707
Median	633.3	35.09	2468	-0.9286	4.376	617.4	15.756	15.667	72711355	14980	0.2911
Mean	671.9	35.11	2698	-0.9340	4.517	653.3	19.558	19.926	75374976	14989	0.2916
3rd Qu.	869.0	47.85	3437	-0.8231	4.899	844.3	24.362	25.270	104361290	16466	0.3125
Max.	1199.7	60.00	6362	-0.3879	17.451	1191.2	110.935	120.437	196036684	17998	0.3333

Subsequently, in Table 2, the correlation between variables of the simulation study was evaluated using Pearson's correlation coefficient matrix.

Table 2: Pearson's correlation coefficient matrix between variables of the simulation study

	C	h	I	CK	μ_Y	o	Q	s	S	sd	CSk	σ_Y	TC	u
C	1.00	-0.02	0.03	-0.00	0.06	0.02	0.06	-0.01	0.03	0.05	-0.01	0.01	0.22	0.14
h	-0.02	1.00	-0.00	0.00	-0.01	-0.00	-0.01	0.02	-0.01	-0.01	0.01	0.01	-0.01	-0.01
I	0.03	-0.00	1.00	-0.31	0.55	-0.01	0.54	0.00	0.73	0.05	0.43	-0.67	0.12	-0.22
CK	-0.00	0.00	-0.31	1.00	-0.02	-0.01	-0.01	0.01	-0.29	0.19	-0.25	0.39	-0.00	0.01
μ_Y	0.06	-0.01	0.55	-0.02	1.00	-0.01	1.00	0.01	0.55	0.83	0.03	-0.04	0.23	-0.38
o	0.02	-0.00	-0.01	-0.01	-0.01	1.00	-0.01	0.00	-0.02	-0.01	-0.01	-0.00	0.01	0.01
Q	0.06	-0.01	0.54	-0.01	1.00	-0.01	1.00	0.01	0.51	0.84	0.01	-0.01	0.24	-0.38
s	-0.01	0.02	0.00	0.01	0.01	0.00	0.01	1.00	-0.00	0.01	-0.00	0.00	-0.01	-0.01
S	0.03	-0.01	0.73	-0.29	0.55	-0.02	0.51	-0.00	1.00	0.06	0.43	-0.64	0.11	-0.20
sd	0.05	-0.01	0.05	0.19	0.83	-0.01	0.84	0.01	0.06	1.00	-0.27	0.49	0.21	-0.32
CSk	-0.01	0.01	0.43	-0.25	0.03	-0.01	0.01	-0.00	0.43	-0.27	1.00	-0.55	-0.00	-0.01
σ_Y	0.01	0.01	-0.67	0.39	-0.04	-0.00	-0.01	0.00	-0.64	0.49	-0.55	1.00	0.01	0.01
TC	0.22	-0.01	0.12	-0.00	0.23	0.01	0.24	-0.01	0.11	0.21	-0.00	0.01	1.00	0.75
u	0.14	-0.01	-0.22	0.01	-0.38	0.01	-0.38	-0.01	-0.20	-0.32	-0.01	0.01	0.75	1.00

An extract of correlations of interest for this study according to their significance given by p-value, are showed in Table 3

Table 3: Extract of correlations of interest for this study according to their significance given by p-value

Relationship	Type	p-value
I vs CK	Inverse	$<.0001$
I vs μ_Y	Direct	$<.0001$
I vs S	Direct	$<.0001$
I vs CSk	Direct	$<.0001$
S vs CK	Inverse	$<.0001$
S vs μ_Y	Direct	$<.0001$
S vs CSk	Direct	$<.0001$

Table 4 show results of linear regression between variables with significant correlations. Note that the inventory stock (I) is lower when the coefficient of kurtosis (CK) takes more positive values. Also, the higher the mean DPUT (μ_Y), the more inventory stock (I) remains. For its part, the symmetry coefficient (CSk) is directly related to the inventory balance in stock (I), the latter being higher when the coefficient takes more positive values. On the other hand, more negative kurtosis coefficients (CK) generate more shortages (S). Along with increasing I, the larger μ_Y, the larger will also be S. This relationship is important to consider since according to the adjusted R-squared value shown in the table 4, 29.82 % of the shortage is explained by μ_Y. Finally, positive coefficients of CSk generate shortages. However, negative values of CSk result in negative degrees of shortage, which is interpreted as over-balance stock.

In order to form groupings of scenarios that are homogeneous among themselves, but in turn heterogeneous among groups of scenarios, a hierarchical cluster analysis was carried out,

Table 4: Results of linear regression between variables with significant correlations.

Dependent variable	Independent variable	Intercept	Slope	Adjusted R-squared
I	CK	427.348	-51.289	0.0974
I	μ_Y	-6338260	0.03006	0.3048
I	CSk	527.786	355.662	0.1890
S	CK	432.582	-51.649	0.0869
S	μ_Y	-13.702.134	0.03169	0.2982
S	CSk	546.998	372.328	0.1827

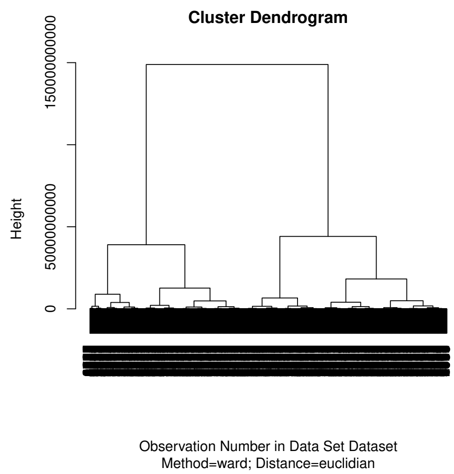

Figure 1: Hierarchical cluster analysis of variables in the simulation study.

which associated the simulated scenarios according to the similarity of multiple variables. This grouping was illustrated in a dendrogram presented in Figure 1

The dendrogram allows us to visualize the variables with which the scenarios were formed in 4 groups. Subsequently, a cluster analysis called "k-means" was carried out, which quantified the number of scenarios by groups, in addition to averaging their variables, thus facilitating data analysis. To confirm this, it is necessary to study the variability of the groups given by the sum of squares, resulting in the intragroup variability $9.948302e^{17}$ and the intergroup variability $1.099817e^{19}$. The intergroup variability is greater than the intragroup variability, which shows us that there are effectively 4 different groups of data. The mean of the variables of the groups of variables of conformation of the scenarios is shown in Table 5.

Table 5: Mean of the variables of the groups of variables of conformation of the scenarios.

Group	I	CK	μ_Y	Q	S	sd	CSk	TC
1	19.14	4.53	648.93	631.16	19.24	2597.06	-0.93	66597107
2	18.18	4.50	621.31	604.12	18.52	2493.29	-0.93	28205097
3	19.79	4.52	695.12	675.42	20.76	2796.15	-0.92	103328543
4	23.25	4.49	794.72	773.61	23.06	3209.21	-0.93	147193441

Table 5 shows an important finding when comparing the averages of the variables, group 4 is the one with the highest total cost, along with the most negative symmetry coefficient and the lowest kurtosis coefficient. It also has the largest shortage and inventory stock. It is also the group with the highest demand and quantity to buy. Group number 2, on the other hand, has a lower total cost, higher kurtosis coefficient, lower shortage, demand and therefore quantity to buy and higher symmetry coefficient. Finally, the probability that a scenario belongs to one of the groups was determined using a multinominal logistic regression model. For this, the values of the regression of the coefficients were tabulated in Table 6.

Table 6: Multinominal logistic regression model .

Group	Intercept	CK (slope)	CSk(slope)
2	0.1828067	-0.041315977	0.009013863
3	0.1222597	-0.003733192	0.252487023
4	-0.7170231	-0.059291893	-0.149842434

As a way to better interpret the multinomial logistic regression for the probability of belonging to groups 2 and 4, which presented lower and higher total cost respectively, then in Table 7 we show an evaluation of these probabilities for minimum values and CK and CSk maximums. Given the characteristics of higher total cost, inventory stock and shortage of group 4, the probability of belonging to this group increases when the kurtosis coefficient is higher, however, there is no significant increase in probability when taking a higher symmetry coefficient. Likewise, only the kurtosis coefficient also significantly determines an increase in group 2 membership when this coefficient takes higher values.

Table 7: Probability of belonging to the groups according to the coefficient of symmetry and kurtosis.

Group	Coefficient	valor	p(group)
2	CK	2.92 (min)	0.4851
		17.451 (max)	0.6319
	CSk	-1.72 (min)	0.4589
		-0.38 (max)	0.4559
4	CK	2.92 (min)	0.7072
		17.451 (max)	0.8506
	CSk	-1.72 (min)	0.6151
		-0.38 (max)	0.6585

4 Discussion and conclusions

This study provides a roadmap by using novel stochastic inventory optimization methods for policy-makers to mitigate inventory stockouts of critical supplies while significantly improving their availability.

The scope of this document is open to both the academic world in the area of operations research and to the practical management of decisions carried out in the real world. In comparison with previous work, our results suggest new decision-making patterns such as considering the skewness and kurtosis of data related to demands per unit of time since it has shown its effect on the variance, which interacts with the best lot sizing and of the total cost. More precisely, a predictive pattern of the first and second stage decisions, that is, the expected quantities in stock and shortages for the use of stochastic lot sizing. Our results indicate that the higher total cost of supply and greater shortage are related to demand patterns with more negative symmetry and lower kurtosis.

Regarding future research, this work is expandable to more general models with heteroscedasticity of variance,such as the widely known variants of generalized lineal models, which are very flexible allowing linear and non-linear functional structures. In addition, in line with this work, the methodology to model a time series, such as the generalized autoregressive and moving average (GARMA) model can be explored and its effects over lot sizing decisions. Furthermore, multivariate time series may be also considered. Some of these issues are being analyzed by the authors, whose findings will be reported in future articles.

The future research trend will focus more on cost saving, inventory availability, uninterrupted supply as well as dealing with unexpected/unpredicted change in the demand. Our study can be regarded as a pioneer in this research perspective by investigating the lot sizing decisions through considering the skewness and kurtosis of data.

References

Alavifard, F. (2019). Modelling default dependence in automotive supply networks using vine-copula. *International Journal of Production Research*, 57(2):433–451.

Casella, G. and Berger, R. L. (2002). *Statistical inference*, volume 2. Duxbury Pacific Grove, CA.

Chan, F., Lim, C., and McAleer, M. (2005). Modelling multivariate international tourism demand and volatility. *Tourism Management*, 26(3):459–471.

Deng, C., Miao, J., Ma, Y., Wei, B., and Feng, Y. (2020). Reliability analysis of chatter stability for milling process system with uncertainties based on neural network and fourth moment method. *International Journal of Production Research*, 58(9):2732–2750.

Escribano, A. and Pfann, G. A. (1998). Non-linear error correction, asymmetric adjustment and cointegration. *Economic Modelling*, 15(2):197–216.

Infanger, G. and Morton, D. P. (1996). Cut sharing for multistage stochastic linear programs with interstage dependency. *Mathematical Programming*, 75:241–56.

Lin, E. M., Sun, E. W., and Yu, M.-T. (2020). Behavioural data-driven analysis with bayesian method for risk management of financial services. *International Journal of Production Economics*, page 107737.

Liu, Z. (2004). Refinement of an inequality of grüss type for riemann-stieltjes integral. *Soochow J. Math*, 30(4):483–489.

Raa, B. and Aghezzaf, H. (2005). A robust dynamic planning strategy for lot-sizing problems with stochastic demands. *Journal of Intelligent Manufacturing*, 16:207–213.

Rohmer, J. and Gehl, P. (2020). Sensitivity analysis of bayesian networks to parameters of the conditional probability model using a beta regression approach. *Expert Systems with Applications*, 145:113130.

Rojas, F. and Ibacache-Quiroga, C. (2020). A forecast model for prevention of foodborne outbreaks of non-typhoidal salmonellosis. *PeerJ*, 8:e10009.

Rojas, F., Leiva, V., Wanke, P., Lillo, C., and Pascual, J. (2019). Modeling lot-size with time-dependent demand based on stochastic programming and case study of drug supply in chile. *PloS one*, 14(3).

Rojas, F., Leiva, V., Wanke, P., and Marchant, C. (2015). Optimization of contribution margins in food services by modeling independent component demand. *Revista Colombiana de Estadística*, 38:1–30.

Rojas, F., Wanke, P., Coluccio, G., Vega-Vargas, J., and Huerta-Canepa, G. F. (2020). Managing slow-moving item: a zero-inflated truncated normal approach for modeling demand. *PeerJ Computer Science*, 6:e298.

Sabet, E., Yazdani, B., Kian, R., and Galanakis, K. (2020). A strategic and global manufacturing capacity management optimisation model: A scenario-based multi-stage stochastic programming approach. *Omega*, 93:102026.

Sadeghi, J. and Niaki, S. T. A. (2015). Two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand. *Applied Soft Computing*, 30:567–576.

Shapiro, A., Dentcheva, D., and Ruszczynski, A. (2014). *Lectures on Stochastic Programming: Modeling and Theory*, volume 16. SIAM, Philadelphia, US.

Stasinopoulos, D. and Rigby, R. (2007). Generalized additive models for location, scale and shape (GAMLSS). *Journal of Statistical Software*, 23:1–46.

Stasinopoulos, M., Rigby, B., Akantziliotou, C., Heller, G., Ospina, R., and Stasinopoulos, M. M. (2020). Package gamlss. dist.

Stasinopoulos, M., Rigby, B., Voudouris, V., Heller, G., and De Bastiani, F. (2015). Flexible regression and smoothing: The gamlss packages in r. *GAMLSS for Statistical Modelling. GAMLSS for Statistical Modeling*.

Stasinopoulos, M., Rigby, R., and Akantziliotou, C. (2008). *Instructions on how to use the gamlss package in R*. Available at www.gamlss.org/wp-content/uploads/2013/01/gamlss-manual.pdf.

Wanke, P. (2009). Consolidation effects and inventory portfolios. *Transportation Research Part E: Logistics and Transportation Review*, 45:107–124.

Wanke, P. (2014). Consolidation effects: Assessing the impact of tail dependence on inventory pooling using copulas. *International Journal of Inventory Research*, 2:174–188.

Wanke, P., Ewbank, H., Leiva, V., and Rojas, F. (2016). Inventory management for new products with triangularly distributed demand and lead-time. *Computers and Operations Research*, 69:97–108.

Wanke, P. and Leiva, V. (2015). Exploring the potential use of the Birnbaum-Saunders distribution in inventory management. *Mathematical Problems in Engineering*, Article ID 827246:1–9.

Wanke, P. F. and Saliby, E. (2009). Consolidation effects: Whether and how inventories should be pooled. *Transportation Research Part E: Logistics and Transportation Review*, 45:678–692.

Zhi, B., Wang, X., and Xu, F. (2020). Impawn rate optimisation in inventory financing: A canonical vine copula-based approach. *International Journal of Production Economics*, 227:107659.