An Adaptive Cubature Kalman Filter for Target Tracking in Underwater Sensor Networks

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***Abstract*—The underwater sensor network is currently a hot research field in academia and industry with many underwater applications, such as ocean monitoring, seismic monitoring, environment monitoring, and seabed exploration. Underwater target tracking is a critical component of ocean development. This paper studies the underwater target tracking problem of the wireless sensor network. In practical applications, the core technology of the target tracking algorithm is the filtering algorithm, which directly identifies the accuracy of the target tracking system. Nonlinear filtering is a hot issue in target tracking because feasible projects are mostly non-linear systems. The linearization method used in traditional Kalman filtering has serious shortcomings in tracking accuracy. Therefore, this paper presents the improved cubature Kalman filtering (ICKF) algorithm for underwater target tracking. There is uncertainty in the target movement, an adaptive forgetting factor is given into the cubature Kalman filtering algorithm to directly modify the error covariance to reduce the impact of uncertainties. Then, interactive multi-model technology is introduced to establish the IMMICKF algorithm with multiple states. Compared with other filtering algorithms, the new algorithm can effectively deal with noise-related non-linear target tracking problems and obtain better estimation accuracy. The numerical simulation is given to demonstrate the effectiveness of the IMMICKF algorithm.**

Keywords—underwater sensor network, target tracking, cubature Kalman filtering, adaptive forgetting factor, interacting multiple model

I. Introduction

With the depletion of land resources development, marine development is becoming the focus of research. Compared with traditional research methods, wireless sensor networks consist of sensor nodes which have the functions of sensing, signal processing, and wireless communication. The wireless sensor network is one of the most important technologies for underwater research[1]. The underwater sensor networks are generally applied for geological monitoring, submarine environmental monitoring and environmental[2][3]. Research on underwater target tracking systems based on wireless sensor networks has been a hot issue in recent years[4]. It can be used in various military and civil fields such as coast defense, marine rescue and fish detection[5].

Underwater target tracking is to obtain the relevant information of the target through multiple sensors and estimate the state of the target's motion. To get more detailed information about the moving target, many underwater target tracking algorithms are proposed. Due to the particularity of the environment, there are numerous limitations in the tracking process. In practice, the core of the target tracking algorithm is the filtering algorithm, which directly determines the accuracy of the target tracking system. In 1960, the famous Kalman filter[6] proposed by Kalman greatly promoted the development of modern filtering theory. Kalman filtering (KF) introduces the state-space model into the theory of optimal filtering and gives the optimal estimation solution of a linear system. However, the Kalman filtering algorithm is only suitable for linear systems. In practical engineering applications, numerous systems are not linear, which limits the application of Kalman filtering in nonlinear systems.

To solve the filtering problem of nonlinear systems, many nonlinear filtering algorithms based on Kalman filtering have been put forward. The extended Kalman filter[7] (EKF) algorithm linearizes the nonlinear system and uses a Kalman filter to estimate the state of the target. The nonlinear problem is transformed into a linear problem by using the Taylor expansion formula, and the first-order term of the Taylor expansion term is retained to reduce the error. The unscented Kalman filtering[8] (UKF) algorithm is based on the unscented transformation (UT) to deal with the nonlinear problem. The UKF does not need to linearize the problem and calculate the Jacobian matrix that the algorithm obtains estimated values by the unscented transformation. The particle filtering[9] (PF) algorithm is implemented by the non-parametric Monte Carlo simulation method. Randomly sampled particles are used to approximate the posterior probability density, which can be utilized to nonlinear and non-Gaussian systems. In 2009, Arasaratnam and Haykin proposed a cubature Kalman filtering (CKF) algorithm[10]. The CKF algorithm uses spherical-radial cubature rule to select sampling points and uses integrals to calculate problems that provide a systematic solution for high-dimensional nonlinear filtering problems.

Some researchers have studied underwater target tracking based on filtering algorithms. The filtering algorithms in target tracking can well estimate and foresee the state of the target to eliminate the target-related uncertainties caused by noise in the networks. The authors[11] implement underwater target tracking based on particle filtering that achieves real-time and stable target tracking in the real environment. The authors[12] design the strong tracking filter based on UKF to deal with correlated noise and sudden change of state. The algorithm adjusts the state prediction error covariance matrix in real-time by introducing a sub-optimal adjustment factor to adjust the gain matrix of the corresponding filter effectively. Compared with other non-linear filtering algorithms such as EKF and UKF algorithms, the cubature Kalman filtering algorithm[13] has the advantages of easy implementation, strong non-linear approximation characteristics and high estimation accuracy. Therefore, CKF is more suitable for the practical environment and is widely used in estimation problems in various fields.

Most of these filtering algorithms are intended for a single motion model of the target. Generally, the underwater target does not perform only one motion mode. Therefore, we focus on multiple models for target tracking in underwater sensor network researchers have conducted related research[14]–[16]. An interacting multiple model (IMM) filter[16] is applied to estimate the position and velocity of the underwater target. The authors[14] propose a centralized fusion algorithm based on interacting multiple models and adaptive Kalman filter for target tracking in underwater acoustic sensor networks. Specifically, by introducing an optimal centralized fusion Kalman filtering algorithm in the adaptive forgetting factor. The algorithm combines the advantages of the Kalman filtering algorithm and the interacting model to implement an interactive Kalman filtering tracking algorithm. In high-dimensional cases, numerical stability and filtering accuracy of the CKF algorithm are superior to the Kalman filtering algorithm. Therefore, we will consider the CKF algorithm in the adaptive forgetting factor for better underwater target tracking accuracy.

The rest of this paper is organized as follows. Section II introduces the underwater target tracking system model. In Section III, a CKF tracking algorithm based on the forgetting factor is presented, and a complete tracking algorithm is proposed. Finally, the performance of the underwater target tracking system by numerical simulation is evaluated in Section IV.

II. SYSTEM MODEL

A. Target motion model

The target tracking algorithms are based on models, and the specific movement state of the target is described through the models. Target tracking system modeling mainly includes two parts: one is a motion model to describe the behavior of the target; the other is a measurement model for the observation of the target's behavior. Therefore, the mathematical expression of the underwater target tracking models is also divided into two parts: the motion model and the measurement model. The general form of the state-space model is as follows:

$$\begin{cases} X_{k+1} = f(X_k) + w_k = A_k X_k + B_k w_k \\ Y_{k+1} = h(X_{k+1}) + v_{k+1} \end{cases} \tag{1}$$

where X_k and Y_k denote the target state and measured variables at k, respectively; w_k and v_k are independent process noise and measurement Gaussian noise sequences with zero means and covariances Q_k and R_k, respectively; f and h are represented as transfer function; A and B are known matrices. A target is set to move to a three-dimensional plane. Its state X_k is composed of position and velocity, namely, $X_k = [x_k, v_x, y_k, v_y, z_k, v_z]^T$. That (x_k, y_k, z_k) denotes the position coordinate of the target and (v_x, v_y, v_z) represents the velocity in *x*, *y*, and *z* coordination at *k*.

The value of target modeling is to obtain the true motion state of the target which is unknown. Therefore, the choice of target motion model directly affects the accuracy of target tracking and the calculation of subsequent filtering algorithm[17]. The three most common target motion models: constant velocity (CV), constant acceleration (CA) and constant turn (CT) models. For the underwater targets, the most common motion states are CV and CT motion[18].

B. CV model

The CV model assumes that the target moves linearly at a constant speed in which the target speed does not change and the acceleration is zero. However, the acceleration cannot be maintained at zero due to interference in reality. Therefore, the $w_k \in R^3$ is a three-dimensional random acceleration disturbance noise obeying the zero-mean Gaussian distribution. T is the sampling interval (assumed in the sampling time kT). Then, the state transition matrix and the disturbance transition matrix are given as follows[19]:

$$A_k = A_{CV} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_k = B_{CV} = \begin{bmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^2/2 \\ 0 & 0 & T \end{bmatrix}$$

C. CT model

The CT model is a motion model used to simulate a target for making a cornering motion with a constant angular velocity. The angular velocity ω is constant over some time. Similarly, the $w_k \in R^3$ is a three-dimensional random acceleration disturbance noise obeying the zero-mean Gaussian distribution. Then, the state transition matrix and the disturbance transition matrix are given as follows[19]:

$$A_k = A_{CT} = \begin{bmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & \frac{\cos(\omega T) - 1}{\omega} & 0 & 0 \\ 0 & \cos(\omega T) & 0 & -\sin(\omega T) & 0 & 0 \\ 0 & \frac{1 - \cos(\omega T)}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} & 0 & 0 \\ 0 & \sin(\omega T) & 0 & \cos(\omega T) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_k = B_{CT} = B_{CV}$$

D. Measurement model

In this paper, it is assumed that all the sensors are homogeneous and the position coordination is known. Sensors measure the distance to the target by transmitting acoustic pulses. The choice of measurement model depends on the way of the sensors detects the target. The measurement model of the target is given as follows[20]:

$$Y_k = \begin{bmatrix} r_k \\ \theta_k \\ \varphi_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2 + z_k^2} \\ tan^{-1}\frac{y_k}{x_k} \\ tan^{-1}\frac{z_k}{\sqrt{x_k^2 + y_k^2}} \end{bmatrix} + v_k$$

where r_k, θ_k and φ_k denote are the observation distance, azimuth angle, and the elevation angle, respectively.

III. PROBLEM FORMULATION

Underwater target tracking systems are generally non-linear, so their motion state estimation is essentially a non-linear filtering problem. The most important metric in target tracking is tracking accuracy, and the selection of filtering algorithm directly affects the tracking efficiency. In this section, we will introduce the adaptive forgetting factor into the CKF algorithm to adjust the covariance matrix to reduce the bias in dynamic models for better tracking accuracy.

A. *Cubature Kalman Filter*

The CKF algorithm is a novel nonlinear filtering algorithm proposed by Ienkaran Arasaratnam and Simon Haykin in 2009, which solves the problem that Kalman filtering has a poor estimation effect when the state variable is high-dimensional. The CKF algorithm is based on a Gaussian filtering framework. Its core is used a third-order spherical-phase diameter volume rule to approximate the posterior mean and covariance of the nonlinear function. The CKF algorithm considers the Gaussian integration from the point of numerical integration.

In CKF, transforming the integral form into a radial integral spherical form and a third-degree spherical-radial rule are the two most critical steps. Therefore, we will briefly introduce the principle of the CKF algorithm. Consider the general points:

$$I(f) = \int_{R^n} f(x)\exp\left(-xx^T\right)dx \qquad (2)$$

Using spherical radial transformation, let $x = ry$, $yy^T = 1$, and $xx^T = r^2, r \in [0, \infty)$.

$$I(f) = \int_0^\infty \int f(ry) r^{n-1} \exp(-r^2)\, d\sigma(y) dr$$

According to the invariant theory, the simplest form of the third-degree spherical cubature rule is assumed as follows:

$$\int f(ry)\, d\sigma(y) \approx w \sum_{i=1}^{2n} f[u]_i$$

$[u]_i$ get the invariant point set after permutation and significant transformation.

According to the Gaussian quadrature rule and the spherical-radial rule, the standard Gaussian weighted integral can be calculated using the third-degree spherical-radial rule which is given as follows:

$$I_N(f) = \int_{R^n} f(x)N(x; 0, I)dx \approx \textstyle\sum_{i=1}^m w_i f(\xi_i) \qquad (3)$$

where $\xi_i = \sqrt{\frac{m}{2}}[\mathbf{1}]_i$, $w_i = \frac{1}{m}$, $i = 1,2, \cdots, m = 2n$. Therefore, using the cubature point set $\{\xi_i, w_i\}$ to numerically calculate the integral, the CKF algorithm can be obtained. Let $x \sim N(x; \hat{x}, P)$ and $P = SS^T$, we can get

$$I_N(f) = \int_{R^n} f(x)N(x; \hat{x}, P)dx \approx \textstyle\sum_{i=1}^m w_i f(S\xi_i + \hat{x}) \qquad (4)$$

Considering the nonlinear system filtering model as in Formula 1, and applying the cubature rule to the KF algorithm. The CKF algorithm can be obtained as follows.

- Initialization

Step 1: Give the filter initial condition values $\hat{X}_0$, P_0, Q_0, R_0. According to the tracking requirements, step 2 to step 8 are performed cyclically at time $k = 1,2, \cdots$.

- Prediction

Step 2: Compute $\hat{X}_{k-1}$ and error covariance matrix P_{k-1} at time $k-1$, and calculate cubature points according to the cubature rule mentioned above.

$$P_{k-1} = S_{k-1}S_{k-1}^T$$

$$\chi_{i,k-1} = \hat{X}_{k-1} + S_{k-1}\xi_i, \;\; i = 1,2, \cdots, m$$

Step 3: Perform cubature points transformation according to the motion model.

$$\hat{\chi}_{i,k|k-1} = f\left(\chi_{i,k-1}\right)$$

Step 4: Compute the one-step prediction state estimate and the one-step prediction error covariance.

$$\hat{X}_{k|k-1} = \frac{1}{m}\sum_{i=1}^m \hat{\chi}_{i,k|k-1}$$

$$P_{k|k-1} = \frac{1}{m}\sum_{i=1}^m \hat{\chi}_{i,k|k-1}\hat{\chi}_{i,k|k-1}^T - \hat{X}_{k|k-1}\hat{X}_{k|k-1}^T + Q_{k-1}$$

- Update

Step 5: Cubature points are calculated from the state prediction $\hat{X}_{k|k-1}$ and the prediction error covariance matrix $P_{k|k-1}$ at time *k*.

$$P_{k|k-1} = S_{k|k-1}S_{k|k-1}^T$$

$$\chi_{i,k|k-1} = \hat{X}_{k|k-1} + S_{k|k-1}\xi_i$$

Step 6: Perform the cubature points non-linear transformation according to the measurement model.

$$Y_{i,k|k-1} = h\left(\chi_{i,k|k-1}\right)$$

Step 7: Compute the measurement prediction, the innovation covariance matrix, cross-covariance matrix between state and measurement, and the filter gain.

$$\hat{Y}_{k|k-1} = \frac{1}{m}\sum_{i=1}^m Y_{i,k|k-1}$$

$$P_{y,k|k-1} = \frac{1}{m}\sum_{i=1}^m Y_{i,k|k-1}Y_{i,k|k-1}^T - \hat{Y}_{k|k-1}\hat{Y}_{k|k-1}^T + R_k$$

$$P_{xy,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \chi_{i,k|k-1} Y_{i,k|k-1}^T - \hat{X}_{k|k-1}\hat{Y}_{k|k-1}^T$$

$$K_k = P_{xy,k|k-1} P_{y,k|k-1}^{-1}$$

Step 8: Compute the posterior state estimation and the estimation error covariance matrix at time *k*.

$$\hat{X}_k = \hat{X}_{k|k-1} + K_k(Y_k - \hat{Y}_{k|k-1})$$

$$P_k = P_{k|k-1} - K_k P_{y,k|k-1} K_k^T$$

The CKF algorithm calculates the state estimation and covariance through a set of sampling points based on a non-linear function, which avoids the linearization of non-linear systems. The algorithm is independent and suitable for any non-linear systems[21].

B. An Improved Cubature Kalman Filter

In the nonlinear estimation problem, the model and the nonlinear system cannot be completely matched due to the statistical characteristics of noise, etc., which affects the effect of filtering estimation. In practical applications, filtering divergence may be caused by inaccurate system models. The strong tracking filter (STF) has been proposed which can effectively enhance the robustness of the non-linear algorithm by adjusting the filtering gain and overcome the influence of uncertain factors on the filtering estimation result[22]. However, the traditional STF algorithm requires that the non-linear functions are continuously differentiable, the filtering performance is poor under strong nonlinearity, and the Jacobian matrix needs to be calculated which makes the STF algorithm limited.

In the case of uncertainties or great noise, the error covariance will be increased. Therefore, the authors [18], [21], [23]propose an adaptive forgetting factor to directly modify the error covariance to compensate for the effect of biased dynamic models. In this paper, we will use the forgetting factor into the CKF algorithm to improve the efficiency of underwater target tracking.

The covariance matrix can reflect the uncertainty of the measurement result. The innovation covariance is equal to $\Lambda_k = P_{y,k|k-1}$. The innovation covariance will be increased in the presence of uncertainties. The increased innovation covariance can be estimated as [24]:

$$\Lambda'_{k+1} = \begin{cases} \eta_1\eta_1^T, & k = 0 \\ \frac{\rho * \Lambda'_k + \eta_{k+1}\eta_{k+1}^T}{1+\rho}, & k \geq 1 \end{cases} \quad (5)$$

where $\eta_k = Y_k - \hat{Y}_{k|k-1}$ is the innovation, and $0 \leq \rho \leq 1$ is a weighting factor determined by the prior data or current data. The relationship between Λ'_k and Λ_k can be expressed as:

$$\Lambda'_k = \tau_k \Lambda_k \quad (6)$$

where $\tau_k = max\left\{1, \frac{1}{m} tr(\Lambda'_k \Lambda_k^{-1})\right\}$ is a scalar variable. Here an adaptive forgetting factor will be given to modify the error covariance to reduce the impact of uncertainties. The error covariance can be expressed as

$$P'_{k|k-1} = \lambda_k P_{k|k-1} \quad (7)$$

The relationship between Λ'_k and $P_{y,k|k-1}$ is extended to the improved CKF algorithm:

$$\Lambda'_k = \lambda_k P_{y,k|k-1} \quad (8)$$

where $\lambda_k \geq 1$ is an adaptive forgetting factor[24] that the prediction error covariance matrix will increase due to the uncertainties. Therefore, the forgetting factor improves tracking efficiency by changing the covariance. The λ_k is a scalar, it can be easily obtained by equations (5)-(8) :

$$\lambda_k = max\left\{1, \frac{tr(\Lambda'_k - R_k)}{tr(P_{y,k|k-1} - R_k)}\right\}$$

According to the adaptive forgetting factor and the CKF algorithm, the update process of the CKF algorithm can be improved as:

Step 5:

$$P'_{k|k-1} = \lambda_k P_{k|k-1} = S_{k|k-1} S_{k|k-1}^T$$

$$\chi_{i,k|k-1} = \hat{X}_{k|k-1} + S_{k|k-1}\xi_i$$

Step 7:

$$P_{y,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Y_{i,k|k-1} Y_{i,k|k-1}^T - \hat{Y}_{k|k-1}\hat{Y}_{k-1}^T + R_k$$

$$\Lambda'_k = \lambda_k P_{y,k|k-1}$$

C. IMMICKF Algorithm

The target's motion characteristics remain unchanged during the interval called a non-motorized target, such as uniform linear motion and uniformly accelerated linear motion. However, in practical applications, the target cannot always keep regular movement, and the external disturbance motion characteristics will always change. The interacting multiple model algorithm is widely used in the field of target tracking. This algorithm uses two or more models to describe the possible states during the movement of the target. Then, the system state estimation is performed by effective weighted fusion, which effectively overcomes the single model estimation error problem. Therefore, this paper uses the Interacting Multiple Model (IMM) combined with the improved cubature Kalman filtering algorithm to perform underwater target tracking more accurately.

Due to the underwater moving targets, this paper mainly uses the CV and CT models mentioned in the second section. The selection of the target motion model in the interactive multi-model algorithm is an important process. This paper mainly uses the CV and CT models mentioned in the second section. The IMMICK algorithm uses multiple filters. Each filter corresponds to a different underwater target state-space model. These models are used to describe the corresponding target motion mode. Each filter has different results depending on the target state.

It is assumed that there are s motion states under the target, and s cubature Kalman filters correspond to them. Among them, the target state equation of the i^{th} model is:

$$X_{k+1}^i = A_k^i X_k^i + B_k^i w_k^i$$

In the IMMICKF algorithm, it is assumed that at any time, the i^{th} model is valid at the current time k. At the time $k-1$, each filter obtains a predicted state value and covariance value. Then, combine the corresponding model probability value of each filter at time $k-1$ and Markov probability transition matrix P to obtain the mixed state estimation value and covariance value of each filter at time k.

The probability transition matrix P represents the probability that the underwater target changes from one state-space model to another state-space model. The matrix P is expressed as follows:

$$P=\begin{bmatrix} p_{11} & \cdots & p_{1s} \\ \vdots & \ddots & \vdots \\ p_{s1} & \cdots & p_{ss} \end{bmatrix}$$

where p_{ij} indicates the probability that the target moves from the i^{th} motion model to the j^{th} motion model.

According to the general IMM algorithm and the improved cubature Kalman algorithm, the optimal IMMICKF algorithm is given as follows:

Step 1: Input interacting

The prediction probability of model j which is the normalization constant is given:

$$c_j=\sum_{i=1}^{s} p_{ij}\mu_i(k-1)$$

where $\mu_i(k-1)$ is the probability of model i at time $k-1$.

The mixture probability from model i to model j is:

$$\mu_{ij}(k-1)=\frac{p_{ij}\mu_i(k-1)}{c_j}$$

The hybrid state estimation of model j is:

$$\hat{x}_{0j}(k-1)=\sum_{i=1}^{s}\hat{X}_i(k-1)\mu_{ij}(k-1)$$

The hybrid error covariance of model j is:

$$P_{0j}(k-1)=\sum_{i=1}^{s}\mu_{ij}(k-1)\{P_i(k-1) + [\hat{X}_i(k-1)-\hat{X}_{0j}(k-1)] \times [\hat{X}_i(k-1)-\hat{X}_{0j}(k-1)]^T\}$$

Step 2: Improved Cubature Kalman Filter

Input the values $\hat{X}_{0j}(k-1)$, $P_{0j}(k-1)$ and the underwater target measurement Y_k to get the predicted values $\hat{X}_j(k)$ and $P_j(k)$ through the filter.

Compute one-step prediction:

$$P_{0j}(k-1)=S_j(k-1)S_j^T(k-1)$$

$$\chi_{i,k-1}=\hat{X}_{0j}(k-1)+S_j(k-1)\xi_i,\ \ i=1,2,\cdots,m$$

$$\hat{\chi}_{i,k|k-1}=f(\chi_{i,k-1})$$

$$\hat{X}_j(k|k-1)=\frac{1}{m}\sum_{i=1}^{m}\hat{\chi}_{i,k|k-1}$$

$$P_j(k|k-1)=\frac{1}{m}\sum_{i=1}^{m}\hat{\chi}_{i,k|k-1}\hat{\chi}_{i,k|k-1}^T-\hat{X}_j(k|k-1) * \hat{X}_j^T(\mathrm{k|k-1})+\mathrm{Q}_j(k-1)$$

Use the adaptive forgetting factor to modify the error one-step prediction covariance by the cubature Kalman algorithm, and the cubature points are calculated.

$$P_j'(k|k-1)=\lambda_j(k)P_j(k|k-1)=S_j(k-1)S_j^T(k-1)$$

$$\chi_{i,k-1}=\hat{X}_{0j}(k-1)+S_j(k-1)\xi_i$$

$$Y_{i,k|k-1}=h(\chi_{i,k|k-1})$$

$$\hat{Y}_j(k|k-1)=\frac{1}{m}\sum_{i=1}^{m}Y_{i,k|k-1}$$

$$P_{y,j}(k|k-1)=\frac{1}{m}\sum_{i=1}^{m}Y_{i,k|k-1}Y_{i,k|k-1}^T-\hat{Y}_j(k|k-1) * \hat{Y}_j^T(\mathrm{k|k-1})+R_j(k)$$

$$P_{xy,j}(k|k-1)=\frac{1}{m}\sum_{i=1}^{m}\chi_{i,k|k-1}Y_{i,k|k-1}^T-\hat{X}_j(k|k-1) * \hat{Y}_j^T(k|k-1)$$

where

$$\Lambda_j'(k)=\lambda_j(k)P_{y,j}(k|k-1)$$

$$\lambda_j(k)=max\left\{1,\frac{tr(\Lambda_j'(k)-R_j(k))}{tr(P_{y,j}(k|k-1)-R_j(k))}\right\}$$

Compute the Kalman gain.

$$K_j(k)=P_{xy,j}(k|k-1)P_{y,j}^{-1}(k|k-1)$$

Compute the posterior state estimation and the estimation error covariance matrix at time *k*.

$$\hat{X}_j(k)=\hat{X}_j(k|k-1)+K_j(k)(Y_k-\hat{Y}_j(k|k-1))$$

$$P_j(k)=P_j'(k|k-1)-K_j(k)P_{y,j}(k|k-1)K_j^T(k)$$

Step 3: Model probability update

The calculation of model probability $\Omega_j(k)$ is realized by the likelihood function. The likelihood function of model j is as follows:

$$\Omega_j(k)=\frac{1}{(2\pi)^{n/2}|\Lambda_j'(k)|^{1/2}}\exp\{-\frac{1}{2}v_j^T[\Lambda_j'(k)]^{-1}v_j\}$$

where $v_j=Y_k-\hat{Y}_j(k|k-1)$.

The probability of model j at time k is as follows:

$$\mu_j(k)=\frac{\Omega_j(k)c_j}{c}$$

where $c=\sum_{j=1}^{s}\Omega_j(k)c_j$ is the normalization constant.

Step 3: Output interaction

Based on the model probabilities corresponding to each filter, the pre-measured state value of each filter is calculated by weighting to obtain the total state estimation $\hat{X}_k$ and the total error covariance estimation P_k at time k.

$$\hat{X}_k = \sum_{j=1}^{s} \hat{X}_j(k)\mu_j(k)$$

$$P_k = \sum_{i=1}^{s} \mu_i(k-1)\{P_i(k) + [\hat{X}_i(k-1) - \hat{X}_k][\hat{X}_i(k-1) - \hat{X}_k]^T\}$$

The final output value of the IMM algorithm is not calculated by selecting a corresponding model at each time to perform state estimation. It is obtained by importing multiple target motion models through the filtering algorithm weighted fusion. The complete target tracking algorithm is shown in Algorithm 1.

Algorithm 1

```
Initialization: P, P_0, X_0, Y_0, μ_0, X̂_0
For k = 1,2,⋯
  For j = 1,2,⋯
    Calculate hybrid estimation X̂_0j(k − 1), P_0j(k − 1)
    Calculate one-step prediction
    Forgetting factor λ_j(k) and Kalman gain K_j(k)
    Calculate state estimate of model j at time K
    Total estimation X̂_k and error covariance estimation P_k
  End for
End for
```

IV. Numerical Simulation

In this section, we use MATLAB software for the numerical simulation to prove the performance of the proposed algorithm.

In the numerical simulation, all the sensors are uniformly distributed in the detection area of $1000m \times 1000m \times 1000m$. The coordinates of each sensor are known and the sensor's sensing radius is $300m$. The sampling interval T is assumed to be $1s$. All the sensors are assumed to be homogeneous and the additive noise variance is set to $\sigma^2 = 5$. The target initial position coordinate X_0 is set as $[300; 10; 300; 10; 100; 2]$. The angular velocity of the CT model is $\omega = 0.052 rad/s$. The target motion model is the CV model in $1s - 40s$ and $80s - 100s$. The target motion model is the CT model in $40s - 80s$. The root means square error (RMSE) is used to measure the performance of the underwater target tracking. The RMSE of the proposed IMMICKF algorithm is set as:

$$RMSE(k) = \sqrt{\frac{\sum_{l=1}^{MC} [\left(\hat{X}_k^l(1) - X_k(1)\right)^2 + \left(\hat{X}_k^l(3) - X_k(3)\right)^2 + (\hat{X}_k^l(5) - X_k(5))^2]}{MC}}$$

where MC is the number of Monte Carlo simulation. All simulation results in this section were performed 100 Monte Carlo simulations.

In the proposed algorithm, we use two motion models, including CV and CT models. The Markov chain transition probability matrix controlling the model transformation is given by:

$$P = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$$

The true trajectory of the target motion and the estimated trajectory of the IMMICKF algorithm is shown in Fig. 1. From Fig. 1, we can see that the optimal algorithm can well estimate the target's motion trajectory.

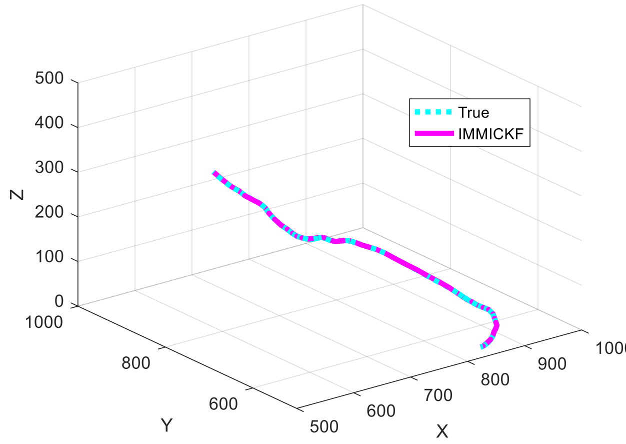
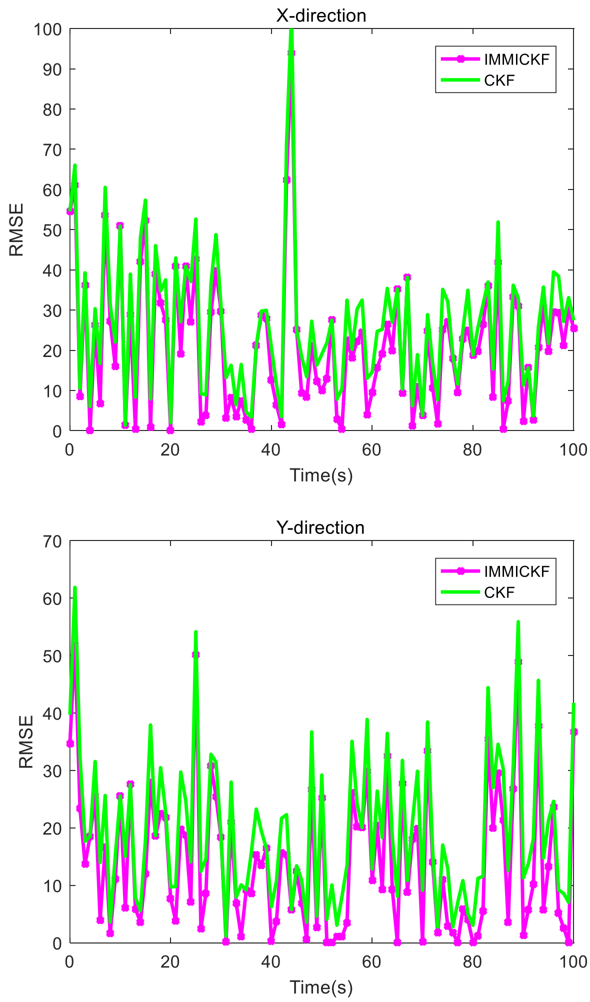


Fig. 1. The true trajectory and the results of the IMMICKF.

Root mean square error is a measure of the deviation between the predicted value and the true value. The RMSEs of positions in X-direction, Y-direction, and Z-direction of the IMMICKF and the CKF algorithm are given in Fig. 2.



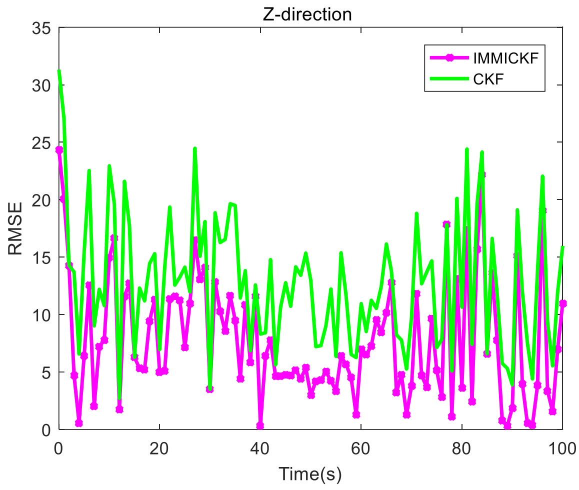


Fig. 2. RMSEs of positions in X-direction, Y-direction, and Z-direction of the IMMICKF and the CKF algorithm.

From Fig. 2, we can see that the RMSEs of positions in X-direction. Y-direction and Z-direction of the IMMICKF algorithm are smaller than the CKF algorithm. When the system model is uncertain, the CKF algorithm cannot adapt to the uncertainty of the model, and the RMSEs of the target motion state is large. The IMMICKF algorithm is utilized to reduce the system uncertainty by introducing an adaptive forgetting factor to complete the underwater target tracking. Meanwhile, compared with the single model CKF algorithm, the proposed algorithm has better tracking accuracy in the Underwater sensor network

V. Conclusions

In this paper, an IMMICKF algorithm has been proposed for underwater target tracking in the underwater sensor network due to the uncertainties of the underwater target tracking system and the traditional CKF algorithm cannot solve the problem of the related noise. Therefore, a new filtering algorithm was established by introducing an adaptive forgetting factor technology to reduce system uncertainty. The optimal ICKF algorithm is proposed based on the optimal CKF algorithm. Then, interactive multi-model technology is introduced to establish the IMMICKF algorithm with CV and CT models because of the variety of motion modes of underwater targets. Finally, compared with the CKF algorithm, simulation results have been summarized as follows: the tracking accuracy of the IMMICKF algorithm is the best in the tracking effect and obtains good estimation error stability. And the proposed algorithm can effectively deal with the non-linear target tracking system. The proposed algorithm can be used in various ocean exploration applications. In future work, we will further study the problems of robustness and computational complexity in the IMMICKF algorithm for target tracking in the underwater sensor network

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