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Abstract This paper explains the thermal behavior of bio-logical tissues exposed to step heating using the Dual-Phase Lag (DPL) bioheat transfer model. This investigation is focused on the effect of thermal-lag times associated with the thermal inertia and microstructural interaction as well as the impact of other thermo-physical parameters such as core body temperature and blood perfusion rate. A Backward Time, Cantered Space (BTCS) implicit scheme is employed for discretization through finite difference method (FDM). Finally, a comparison is proposed be-tween Penne's model, the Thermal wave model, and the DPL model to illustrate the effect of different relaxation times.

Keywords— *BTCS discretization scheme, Dual-Phase Lag model, Finite Difference Method , Penne's bioheat model, Thermal Wave model.*

1. Introduction

Thermal therapy and thermal exposure or risks to the skin have both benefited from a better understanding of heat transmission in biological tissues. Many bioheat transfer models have been developed to understand thermal behavior in living tissues [1]. The most widely used Bio-heat transfer model is Penne's equation which includes blood perfusion rate and metabolic heat generation etc., as the source terms and the thermal behavior was described by classical Fourier's law. This equation has been employed in different numerical analyses of thermal comfort [2], evaluation of burn injuries [3-5], thermal therapy [6-8], and thermal parameter estimation [9-11], etc. According to the Fourier law, the infinite quick transmission of thermal flux stands inconsistent with the practical scenario in living tissues. So a new modified non-Fourier type hyperbolic heat transfer model (Thermal Wave Bioheat Transfer Model) was introduced by Cattaneo and Vernotte [12,13], in which heat flux was taken to propagate like a wave with a finite speed. The thermal wave model was implemented in both homogeneous materials [14-16] with a relaxation time ranging from 10^{-8} to 10^{-14} to observe the thermal wave nature as well as in non-homogeneous biological living tissues to investigate the lag times for heat flux for different tissues [17-19]. Furthermore, another phase relaxation time was incorporated for a temperature gradient to examine the effect of microstructure in the tissues, in order to produce a more realistic solution to the existing thermal wave bioheat transfer model for non-homogeneous and anisotropic living tissues. The dual-phase lag model is a modified equation that includes the combined effect of phase-lag timings for heat flux owing to thermal inertia and temperature gradient due to the influence of microstructure interaction (DPL model) [20,21].

$$T_q \rho C \frac{\partial^2 T}{\partial t^2} = k\left[\nabla.(\nabla T) + T_t \frac{(\partial \nabla.(\nabla T)}{\partial t}\right] + (\rho C)_b G(T_{cr} - T) - \left(T_q G(\rho C)_b + \rho C\right)\frac{\partial T}{\partial t} + Q_m + T_q \frac{\partial Q_m}{\partial t} \quad (1)$$

Here, C, and k denote the density, specific heat, and thermal conductivity of the tissues. G and $(\rho C)_b$ are the blood perfusion rate and total heat capacity of the blood, respectively. T_q and T_t denotes the phase relaxation time for heat flux and temperature gradient, respectively, whereas Q_m is the metabolism heat generation rate. Equation (1) represents the unsteady DPL equation, if T_t is neglected, then the equation behaves as a thermal wave equation, and if both T_q *and* T_t are neglected, then the equation becomes classical Penne's equation. Some experimental [22, 23], analytical [24-26], and numerical [27] investigations were done to find out both phase lag times. Numerous works are being carried out to explore the exact values for phase lag times in different situations.

In this paper DPL model is applied with the step heating heat flux with BTCS discretization scheme using finite difference numerical method. Furthermore, the effect of different parameters like core body temperature (T_{cr}), both phase lag times (T_q and T_t) and blood perfusion rate (G) are investigated. Finally, alternative bioheat transmission models such as Penne's, Thermal Wave, and DPL are compared.

2. Mathematical Formulation and Numerical Modelling

Equation (1) is used as the DPL bioheat model for the current problem. A step heating is applied to the skin having a flux of 40kW/m^2 for 20s, and it is allowed to cool for the following 30s.The metabolism heat generating rate (Q_m) is ignored here because it is negligible in comparison to the applied heat flux. A finite-difference numerical approach is with BTCS implicit scheme is applied to discretize the governing equation. The Tri Diagonal Matrix Algorithm (TDMA) is used to solve all discretized equations. In the MATLAB 2016 solver, a computer code is developed with consistent step sizes for space and time of 0.00008 and 0.01. According to Table-1, the thermo-physical characteristics of skin and blood are obtained. The skin is considered as one layer with the following boundary and initial condition.

A. Initial Conditions:

i. $T(x,0) = 307.15\ K$; $(0 \le x < L)$ (2a)

ii. $\frac{\partial T(x,0)}{\partial t} = 0$; $(0 \le x < L)$ (2b)

B. Boundary Conditions:

i. $Q(0,t) = -k\frac{\partial T}{\partial t}$; (0≤t≤a) (2c)

Where, $Q(0,t) = q_0[u(t) - u(t-a)]$

ii. $T(L,t) = T_{cr}$; (2d)

Where $u(t) = 1$, when $t > 0$, and $u(t) = 0$ when $t < 0$.

Where $u(t)$ = unit step function, q_0 = heat flux during the period of heating, a = heating time duration, L= Total skin thickness and T_{cr} = body core temperature.

Table .1 Thermo-physical properties of skin and blood

Properties / Layer	*Density ρ (Kg/m³)*	*Specific heat C (J/Kg K)*	*Thermal conductivity k (W/m K)*	*Blood perfusion rate G(Kg/m³)*	*Thickness L (m)*	*Body core Temperature T_{cr} (^{0}C)*
Skin [28]	*1000*	*4200*	*0.2,0.5*	*0.5,1.5,3*	*0.01208*	*34 , 37 ,39 ,40*
Blood [29]	*1060*	*3770*	--	--	--	--

The present numerical model is validated with Liu et. al.[28] by taking a constant temperature of 100^{0}C exposed to the skin for 15 s followed by cooling with a temperature of 0^{0}C for the next 30 s as presented in Fig.1, which shows an excellent agreement with an error of less than 5%.

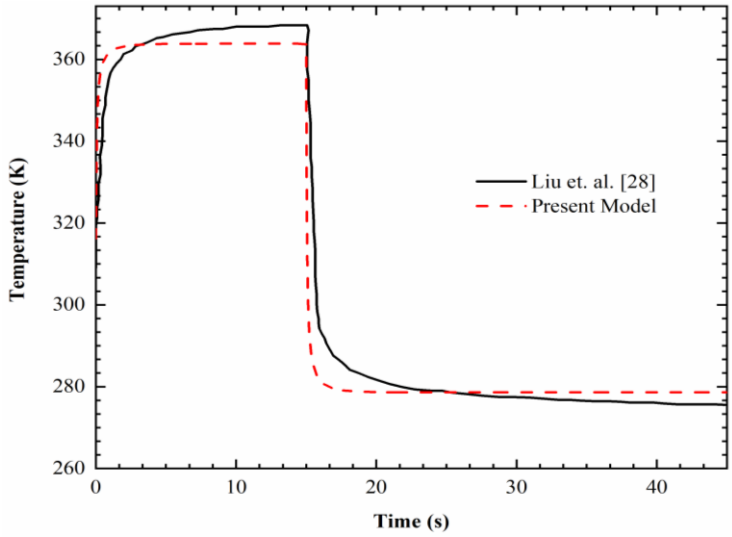

Figure 1. Comparison of temperature variation with the result of Liu et. al [28] at a skin depth of 0.00008 m

3. Result and Discussion

A. Effect of relaxation times for heat flux (T_q) and temperature gradient (T_t) on temperature distribution in tissue :

Relaxation times for heat flux (T_q) and temperature gradient (T_t) are mostly depending on the tissues, blood vessels, and different body. Fig. 2 shows the temperature variation of biological tissue with time taking different relaxation time for heat flux (T_q) and temperature gradient (T_t). Fourier's classical conduction model (Penne's model; T_q=T_t=0) shows a sudden increase in temperature with a steeper slop and flattens quickly to a maximum temperature due to the propagation of heat in an infinite manner. DPL model for the condition $T_q = T_t$ coincides with Penne's model for small values of T_q and T_t (T_q=T_t=0.1) but for the larger phage lag times (T_q=T_t=10), a significant temperature difference is observed. A sudden temperature rise for $T_t > T_q$ (T_q=0.1, T_t=10, and T_q=0.1, T_t=1) is also noticed because the diffusive nature is dominated for heat propagation.A large T_t/T_q value shows more temperature rise. The condition $T_t < T_q$ (T_q=10 and T_t=0.1) shows a comparably less rise in temperature because the wave nature dominates over the diffusive nature of heat propagation. When T_t/T_q tends to or becomes zero (T_q=20 and T_t=0), the DPL model behaves like a hyperbolic thermal wave model, which records the least

temperature. During the cooling phase, all the model converges to Penne's model with a sudden drop in temperature as steady-state is established with time increment. On the other hands, the DPL model having a larger T_t value shows a gradual decrement of temperature and takes more time to converge to the steady-state as heat is more penetrating the skin depth due to the more diffusive nature. The comparison reveals that Penne's model evinces more temperature rise, followed by the DPL and Thermal wave models. However, the DPL model is more realistic as it considers thermal inertia (wave nature of heat propagation) and microstructure interaction of tissue (anisotropic nature of the living tissues). Fig. 3 depicts the temporal changes of tissue temperature for different skin depths(x) for T_q=10 and T_t=0.1.

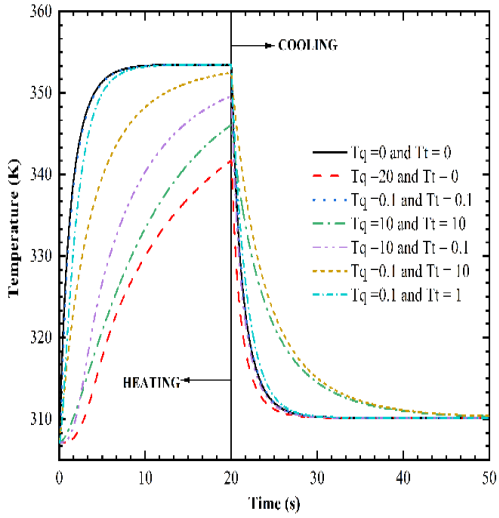

Figure 2. Variation of temperature with time for k = 0.2 W/mK,T_{cr} = 37 ^{0}C at skin depth (x) = 0.00016 m

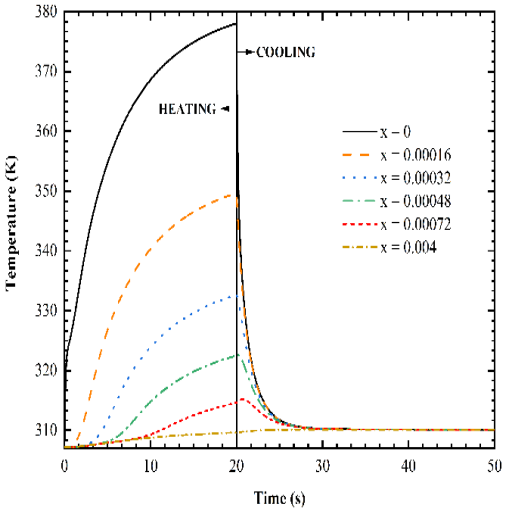

Figure 3. Variation of temperature with time for k = 0.2 W/mK , G = 0.5 Kg/m3,Tq= 10 and Tt = 0.1 at different skin depth (x) in m

B. Effect of body core temperature*:*

Human body core temperature varies depending upon deferent conditions like fever, hypothermia, hyperthermia, and hyperpyrexia [30]. So it is significant to observe the effect of body core temperature on thermal regulation of living tissues. Fig. 4 shows the effect of four different core body temperature (34^{0}C, 37^{0}C, 39^{0}C, 40^{0}C) on tissue temperature at two different skin depth (0.00008m and 0.00016m). It is observed that a high body core temperature leads to more temperature rise and becomes stable at a higher temperature with the increment of time, as larger core temperature ensues comparatively less temperature gradient in the tissue resulting less heat propagation towards body core. Hence more heat will be accumulated in the tissue resulting a temperature rise.

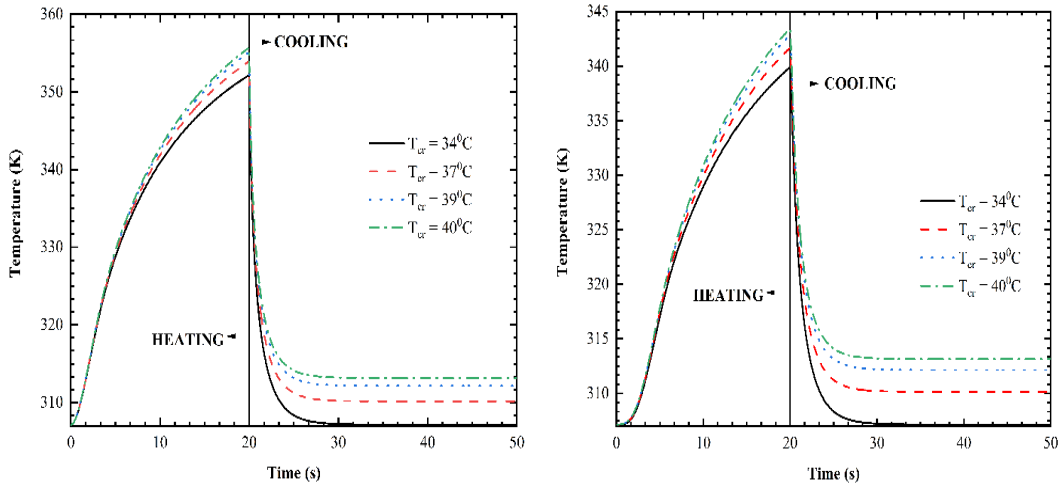

Figure 4. Variation of temperature with time for k = 0.2 W/mK , G = 0.5 Kg/m^3, T_q = 20 and T_t = 0.05 at skin depth (x) (a) 0.00008 m and (b) 0.00016 m

C. Effect of blood perfusion rate on temperature distribution in tissue:

In the present study, three different perfusion rates (0.5, 1.5 and 3 Kg/m^3) are also taken at two different skin depths (0.00008 m and 0.00016 m) to observe the effect on tissue temperature rise. Blood perfusion rate acts as a cooling function for the rising rate of tissue surface temperature, as the heat is taken away by the blood due to temperature difference between tissue surface temperature and body core temperature. It is observed from Fig. 5 that a larger blood perfusion rate prevents the surface temperature rise of the tissues in a shorter time. It is also noticed that a larger perfusion rate takes less time to attend a steady-state, which means the slope of the temperature drop is less in the case of the larger perfusion rate. The temperature drop for the larger perfusion rate is more sudden than the lower perfusion rate.

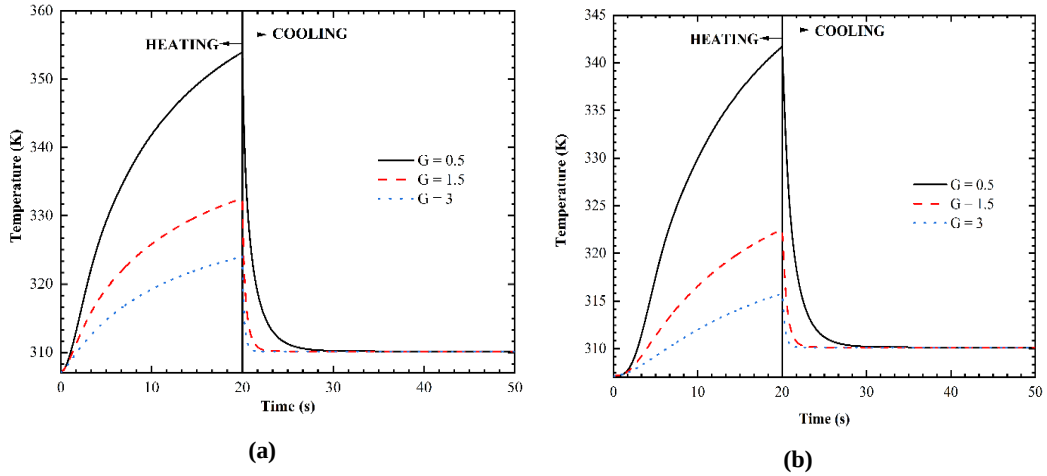

Figure 4. Variation of temperature with time for k = 0.2 W/mK,T_{cr} = 37 ^{0} C, T_q = 20 and T_t = 0.05 at skin depth (x) (a) 0.00008 m and (b) 0.00016 m

4. Conclusion

Numerical analysis is conducted to observe the influence of the DPL model, blood perfusion rate, and body core temperature on a single layer of skin subjects to a step heating of 40 kW/m^2 for 20 seconds heating followed by 30 seconds cooling. Temperature rise is found more in the case of the dominant diffusive nature of the heat propagation ($T_t > T_q$), and thermal inertia dominated heat propagation ($T_t < T_q$) shows a wave-like nature that drops the temperature rise of the living tissue. Blood perfusion rate plays a cooling function for the tissue surface temperature. Temperature rise drops and attains a steady state at a lower time for a larger perfusion rate. Core body temperature is found to be a crucial parameter that affects tissue temperature more prominently. A larger core temperature shows a higher temperature rise.

References

[1] Arkin, H., Xu, L. X., & Holmes, K. R. (1994). Recent developments in modeling heat transfer in blood perfused tissues. IEEE Transactions on Biomedical Engineering, 41(2), 97-107.

[2] Arens, E., & Bosselmann, P. (1989). Wind, sun, and temperature—Predicting the thermal comfort of people in outdoor spaces. Building and Environment, 24(4), 315-320.

[3] Su, Y., He, J., & Li, J. (2016). Modeling the transmitted and stored energy in multilayer protective clothing under low-level radiant exposure. Applied Thermal Engineering, 93, 1295-1303.

[4] Song, G., Barker, R. L., Hamouda, H., Kuznetsov, A. V., Chitrphiromsri, P., & Grimes, R. V. (2004). Modeling the thermal protective performance of heat-resistant garments in flash fire exposures. Textile Research Journal, 74(12), 1033-1040.

[5] Su, Y., Li, R., Yang, J., Song, G., & Li, J. (2020). Effect of Compression on Contact Heat Transfer in Thermal Protective Clothing Under Different Moisture Contents. Clothing and Textiles Research Journal, 38(1), 19-31.

[6] Shrestha, D. C., & Acharya, S. (2021). Time-Dependent Mathematical Model of Thermoregulation in Human Dermal Parts During Sarcopenia. Journal of Nepal Mathematical Society, 4(1), 41-53.

[7] Deng, Z. S., & Liu, J. (2002). Analytical study on bioheat transfer problems with spatial or transient heating on the skin surface or inside biological bodies. J. Biomech. Eng., 124(6), 638-649.
[8] Kengne, E., Mellal, I., & Lakhssassi, A. (2014, October). Bioheat transfer problems with spatial or transient heating on the skin surface or inside biological bodies. In 2014 7th International Conference on Biomedical Engineering and Informatics (pp. 792-797). IEEE.
[9] Chato, J. C. (1980). Measurement of thermal properties of growing tumors. Annals of the New York Academy of Sciences, 335(1), 67-85.
[10] Liu, J., & Xu, L. X. (1999). Estimation of blood perfusion using phase shift in temperature response to sinusoidal heating at the skin surface. IEEE Transactions on Biomedical Engineering, 46(9), 1037-1043.
[11] Valvano, J. W., Allen, J. T., & Bowman, H. F. (1984). The simultaneous measurement of thermal conductivity, thermal diffusivity, and perfusion in small volumes of tissue.
[12] Cattaneo, C. (1958). A form of heat conduction equations which eliminates the paradox of instantaneous propagation. Comptes Rendus, 247, 431.
[13] Vernotte, P. (1958). Les paradoxes de la theorie continue de l'equation de la chaleur. Compt. Rendu, 246, 3154-3155.
[14] A.V. Luikov, Analytical Heat Diffusion Theory, Academic Press, New York, 1968, pp. 245–248.
[15] Kaminski, W. (1990). Hyperbolic heat conduction equation for materials with a non-homogeneous inner structure.
[16] Brazhnikov, A. M. (1975). One engineering method of calculating heat conduction processes. Inzhenerno Fizicheskij Zhurnal, 28, 677.
[17] Mitra, K., Kumar, S., Vedevarz, A., & Moallemi, M. K. (1995). Experimental evidence of hyperbolic heat conduction in processed meat.
[18] Roetzel, W., Putra, N., & Das, S. K. (2003). Experiment and analysis for non-Fourier conduction in materials with non-homogeneous inner structure. International Journal of Thermal Sciences, 42(6), 541-552.
[19] Herwig, H., & Beckert, K. (2000). Experimental evidence about the controversy concerning Fourier or non-Fourier heat conduction in materials with a non-homogeneous inner structure. Heat and Mass Transfer, 36(5), 387-392.
[20] O¨ zis¸ ik, M. N., & Tzou, D. Y. (1994). On the wave theory in heat conduction.
[21] Tzou, D. Y. (1995). A unified field approach for heat conduction from macro-to micro-scales.
[22] Mitra, K., Kumar, S., Vedevarz, A., & Moallemi, M. K. (1995). Experimental evidence of hyperbolic heat conduction in processed meat.
[23] Banerjee, A., Ogale, A. A., Das, C., Mitra, K., & Subramanian, C. (2005). Temperature distribution in different materials due to short pulse laser irradiation. Heat Transfer Engineering, 26(8), 41-49.
[24] Rukolaine, S. A. (2014). Unphysical effects of the dual-phase-lag model of heat conduction. International Journal of Heat and Mass Transfer, 78, 58-63.
[25] Zubert, M., Raszkowski, T., Topilko, J., Starzak, L., Jablonski, G., Janus, P., & Napieralski, A. (2018, June). Determining Parameters of the Dual-Phase-Lag Model of Heat Flow. In 2018 25th International Conference" Mixed Design of Integrated Circuits and System"(MIXDES) (pp. 225-230). IEEE.
[26] Lin, J., Zhang, Y., & Reutskiy, S. (2021). A semi-analytical method for 1D, 2D and 3D time fractional second order dual-phase-lag model of the heat transfer. Alexandria Engineering Journal, 60(6), 5879-5896.
[27] Liu, K. C., Wang, Y. N., & Chen, Y. S. (2012). Investigation on the bio-heat transfer with the dual-phase-lag effect. International Journal of Thermal Sciences, 58, 29-35.
[28] Liu, K. C. (2008). Thermal propagation analysis for living tissue with surface heating. International Journal of Thermal Sciences, 47(5), 507-513.
[29] Liu, J., Chen, X., & Xu, L. X. (1999). New thermal wave aspects on burn evaluation of skin subjected to instantaneous heating. IEEE transactions on biomedical engineering, 46(4), 420-428.
[30] https://en.wikipedia.org/wiki/Fever#Hyperpyrexia.