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Production and Energy Utilization of Landfill
Gas in the Pelagonija Region in Republic of N.
Macedonia

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Mathematical Model for Assessment of Production and Energy Utilization of Landfill Gas in the Pelagonija Region in Republic of N. Macedonia

Blagoj Dimovski¹, Cvete Dimitrieska², Sonja Calamani³ and Gordana Janevska⁴

Abstract – This paper presents a simulation of biogas production in the projected landfill in the Pelagonija region in the Republic of N. Macedonia, using the Landfill Gas Emissions Model (LandGEM, version 3.03). The purpose of the analysis is to obtain the amount of produced biogas and the possibility of using it as energy, as well as the other products from anaerobic digestion which have a negative impact on the environment.

Keywords – Anaerobic digestion, Biogas, LandGEM model, Landfill.

I. INTRODUCTION

Municipal solid waste management requires a partial analysis of many aspects to establish a system with sustainable development. Many mathematical models have been made that represent a different way in which the collection, selection, transport, and storage of municipal solid waste is designed [1], [2]. The developed plan for the Pelagonija region foresees the reduction of organic biodegradable waste that is taken to the landfill, and it is directed to processes of recycling, composting, etc. However, a significant part of the biodegradable waste will end up in the landfill where biogas will be produced through anaerobic digestion over time. This enables an installation of energy systems for utilizing the components of biogas (methane), as well as preventing negative impacts on the environment (release of carbon dioxide and NMOC – nonmethane organic compounds). Therefore, a mathematical model and simulation for the production of biogas in the planned landfill is made using LandGEM [3], based and adapted according to the projected analyzes for the quality, quantity and composition of the municipal solid waste in the Pelagonija region. The Landfill Gas Emissions Model (LandGEM, version 3.03) tool is available for free use.

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II. MATHEMATICAL MODEL

Gas production in landfills is estimated through several models that are adapted according to different aspects, such as: ADM1 model that includes several kinetic and stoichiometric parameters [4], [5], AMOCO model where the influence of water on anaerobic digestion is considered, the morphological composition of the waste [6], etc.

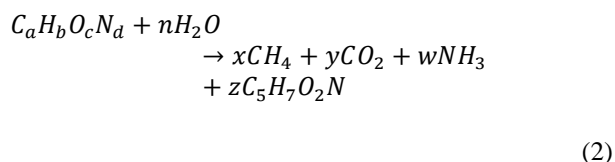
The LandGEM model was selected as the most appropriate model for estimating biogas emissions, which uses parameters for waste entering the landfill, landfill capacity, active operational period and approximates parameters for waste composition and moisture. Accordingly, LandGEM calculates the annual biogas production through input parameters for operation of the landfill and the amount of landfilled waste. Eq. (1) of first-order decomposition rate for calculating the annual production of CH_4 is:

$$Q_{CH_4} = \sum_{i=1}^n \sum_{j=0.1}^i kL_0 \left(\frac{M_i}{10}\right) e^{-kt_{ij}} \quad (1)$$

where:

- Q_{CH_4} - annual methane generation in the year of the calculation ($m^3/year$)
- i - 1 year time increment
- n - (year of the calculation) - (initial year of waste acceptance)
- j - 0.1 year time increment
- k - methane generation rate ($year^{-1}$)
- L_0 - potential methane generation capacity (m^3/Mg)
- M_i - mass of waste accepted in the i^{th} year ($Mg = t$)
- t_{ij} - age of the j^{th} section of waste mass M_i accepted in the i^{th} year (decimal years, e.g. 3.2 years) [3]

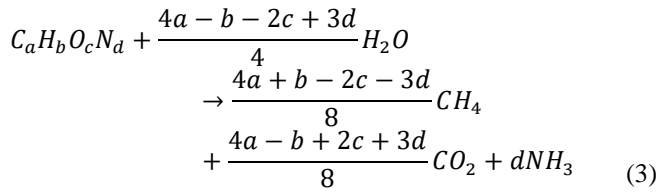
L_0 is calculated according to the chemical equation Eq. (2), Andreotto and Cossu (1988), which is defined as first-order reaction and that considers the influence of structural and operational landfill factors on the anaerobic digestion [7], [8].



where:

- n, x, y, z, w - stoichiometric coefficients of the reaction molecules.

$C_5H_7O_2N$ is a molecule of the bacterium that is neglected over time, resulting in Eq. (3):



where:

a, b, c, d - number of atoms in the elements of molecules.

From Eq. (3) we get the relation that is valid under normal conditions:

$$1 \text{ mol } C = 1 \text{ mol biogas } (CH_4 + CO_2) \gg 1 \text{ kg} = 1.867 \text{ m}^3 \text{ biogas}$$

Given that biogas production directly depends on the amount of biodegradable organic waste, Eq. (4) is used [7]:

$$C_0 = 1.867 \cdot \sum_i (C_e)_i = 1.867 \cdot \sum_i [(C)_i \cdot (fb)_i \cdot (1 - u)_i \cdot (p)_i] \quad (4)$$

where:

- C_0 - potential capacity for biogas production, $L_0 = C_0/2$ [m^3/kg] in biogas production of 50%.
- $(C_e)_i$ - amount of biodegradable organic carbon in component i [kg/kg waste];
- $(C)_i$ - amount of organic carbon in component i [kg/kg];
- $(fb)_i$ - fraction (biodegradable) of $(C)_i$ [kg/kg];
- $(u)_i$ - moisture content in the component [kg water / kg of wet weight of component i];
- $(p)_i$ - mass of wet component i .

Due to the specificity of the waste and the conditions that are expected to prevail, parameters are defined in Tables I and II [9] that deviate from the classic parameters prescribed in [3], [7]:

TABLE I
PARAMETERS FOR MIXED WASTE

Components	$(u)_i$	$(C)_i$	$(fb)_i$
Biodegradables	0.65	0.6	0.85
Paper - Cardboard	0.25	0.4	0.5
Others	0.3	0.55	0.2

TABLE II
VALUES OF WASTE COMPONENTS

Components	Rate of biodegradation	Halftime	k	L_0
Biodegradables	Short term	3.75	0.185	167
Paper - Cardboard	Medium term	6.93	0.100	140

Others	Long term	23.10	0.030	72
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The calculation is performed according to the waste management plan for the Pelagonija region, where the landfill is expected to operate for 25 years ($n=25$), thus the calculations determine a period between 2024-2049 in the active operation of the landfill, where extraction of biogas is 60%. During the rehabilitation period after the closure of the landfill (>2049), biogas extraction of 75% is predicted.

Table III shows a projection of the composition of the biodegradable waste, where the section "others" actually represents the compensation of error in the predicted waste shares.

TABLE III
PROJECTED COMPOSITION SHARE OF BIODEGRADABLE WASTE

Residues composition	%
Organic	59.1%
Paper/Cardboard	39.4% [^]
Others	1.5%

III. RESULTS

According to the selected parameters in Eq. 1 which are adapted to the conditions in the Pelagonija region, a simulation is made in LandGEM for the change in the parameters of the anaerobic digestion products during the active period and the rehabilitation period after the closure. The results show the change of methane (CH_4) as an energy potential, carbon dioxide (CO_2) and other organic components (NMOC) as a function of time.

The results are shown in Figs. 1 and 2, while Fig. 3 represents the production of biogas and its energy utilization according to the plan for the Pelagonija region [9].

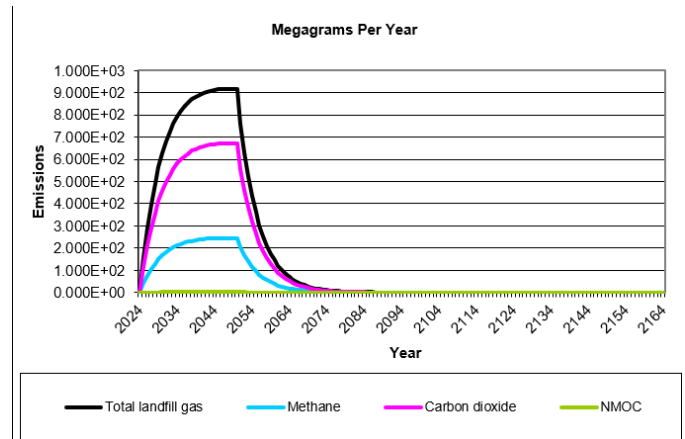


Fig. 1. Production of pollutants in $Mg/year$ (megagram = tone)

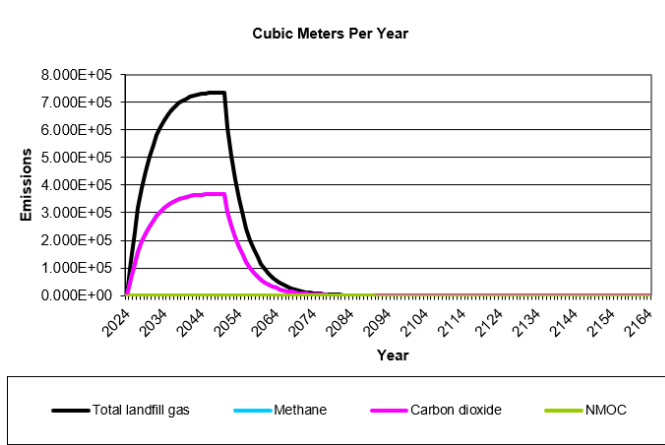


Fig. 2. Production of pollutants in $m^3/year$

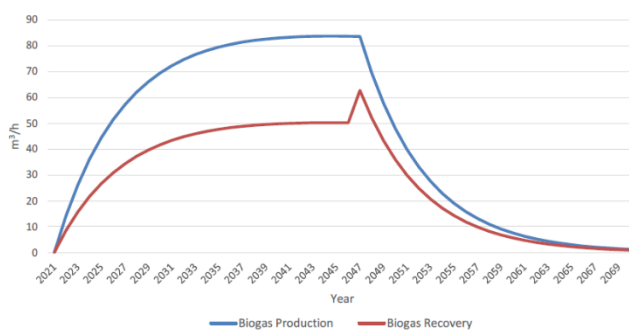


Fig. 3. Production of biogas and its utilization

IV. CONCLUSION

The analysis of the process of anaerobic digestion to predict the production of biogas and other products in the landfill provides an opportunity for planning and designing systems for energy utilization and protection of the environment from harmful effects (smells, discharge, organic and inorganic gases). The obtained results show that the maximum production of biogas occurs during the landfill closure period, but significant emissions also occur during the rehabilitation of the landfill. The predicted capacity of $100 Nm^3/h$ will satisfy the needs of the landfill as the maximum value reaches $83.6 Nm^3/h$.

REFERENCES

[1] J. Valizadeh, P. Mozafari, A. Hafazalkotob, "Municipal Waste Management and Electrical Energy Generation from Solid Waste; a Mathematical Programming Approach", *Journal of Modelling in Management.*, vol. 17, no. 1, pp. 309--340, 2022.

[2] M. Barma, H. K. Beniyamin, U. M. Gaya, "Mathematical Model of the Optimization of Municipal Solid Waste Management", *Frontiers in Sustainability.*, vol. 3, no. 1, pp. 18, 2022.

[3] A. Alexander, C. Burkin, A. Singleton, "Landfill Gas Emissions Model (LandGEM) Version 3.02 User's Guide". <https://www3.epa.gov/ttnecat1/dir1/landgem-v302-guide.pdf>

[4] C. Manjusha, B. Beevi Sajeena, "Mathematical Modeling and Simulation of Anaerobic Digestion of Solid Waste", *Procedia Technology* 24, no. 1, pp. 654--660, 2016.

[5] L. Delgadillo-Mirquez, M. Hernandez-Sarabia, M. Machado-Higueria, "Mathematical Modeling and Simulation for Biogas production from Organic Waste", *International Journal of Engineering Systems Modelling and Simulation.*, vol. 10, no. 2, pp. 97--102, 2018.

[6] M. Temirbekova, M. Aliyarova, I. Iliev, A. Yelemanova, S. Sagintayeva "The Generation of a Mathematical Model of the Biogas Production Process from Organic Municipal Solid Waste", *E3S Web of Conferences.*, vol. 180, p. 02019, 2020.

[7] G. Mannina, D. Di Trapani, G. Viviani, "Mathematical Modeling of Biogas from Municipal Solid Waste Landfill", 2016. <https://www.semanticscholar.org/paper/Mathematical-modelling-of-biogas-from-municipal-Mannina-Trapani/c1d5d66a2d66f483268475bfd89a629ea64bd4c4>

[8] B. B. Milutinovic, G. M. Stefanovic, K. Z. Dencic-Mihajlov, P. S. Djekic, A. D. Borcic "Mathematical Model for Evaluation of Cost-Effectiveness of Waste Treatment Technique with Energy Recovery", *Thermal Science.*, vol. 20, no. 5, p.p. 1573--1584, 2016.

[9] https://www.moep.gov.mk/wp-content/uploads/2014/11/FSCBA_PelagonijaR_112017.pdf