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Composite Labelling of Some Graphs and Application

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COMPOSITE LABELLING OF SOME GRAPHS AND APPLICATION

1. ABSTRACT

The variables are $u, v, w \in V(H)$. A composite labelling is a bijection f : $(V(H))\bigcup(E(H)) \rightarrow 1, 2, 3, ..m + n$ in which $\gcd(f(uv), f(vw)) \neq 1$.Composite Graph is a graph that states composite labelling. We will use the Star graph $k(1, n)$ to integrate networking, block chain, and online commerce in this study. In addition we show that Composite labelling admit in Star graph, The crown graph $(c_n \times k_1)$,The comb graph $(P_n \times K_1)$, The Bistar graph $B_{(n,n)}$, Join sum of two copies of cycle(c_n), Two copies of even cycles$c_n (n \geq 6)$ partaking a common edge, One point union of six copies of p_4 , All caterpillar trees, Flower graph $f_{(n\times m)}$.

Keywords: Composite Labelling, Star Graph, Cycle Graph, Comb Graph.

2. Introduction

Let H be a finite, directionless, and simple graph. Let $V(H)$ indicate the vertices set of a graph H and $E(H)$ indicate the edges set.A graph labelling is the process of assigning labels to edges and/or vertices of a graph, which are conventionally represented by integers. A vertex labelling is a function of V to a collection of labels in a fixed graph $H = (V, E)$; a graph containing such a function is called a vertex-labeled graph. An edge labelling is a function of E to a collection of labels in the same way. The graph is known as an edge-labelled graph in this scenario. For various graph labelling problems, we refer to Gallian[1]. The concept of composite labelling was first introduced by Kureethara Joseph Varghese and p.Stephy Maria[2]. The composite labelling for tree, cycle, and ladder graphs has been shown by Kureethara

Joseph Varghese and P.Stephy Maria. In this study, we look into composite labelling of particular graph types and how it might be used.

3. PRELIMINARY

3.1. **Definition.** "H (V, E) is a directedless simple connected graph with order n of size m. Let uv and vw be two edges that intersect the shared vertex v. uvw is a two-length route in the graph H. $f : (V(H) \bigcup E(H)) \rightarrow 1, 2, 3, ..., m + n$ is a bijective function with the condition that gcd $(f(uv), (vw)) \neq 1$. On composite graphs, the labels are also composite."

3.2. **Definition.** "If it has no loops or parallel edge then the graph is called simple, where loop is an edge with identical ends and two or more lines with the same pair of edges are similar edges".

3.3. **Definition.** "A star graph is a complete bipartite graph with just one vertex in one partition and one or more in the other."

3.4. **Definition.** "Joining a pendant edge to each vertex of cycle yields the crown graph $(C_n o K_1)$".

3.5. **Definition.** "A comb graph is created by connecting each vertex of a route with a single pendant edge."

3.6. **Definition.** "Bistar diagram, The graph $B_{(n,n)}$ is created by linking the apex vertices of 2 copies of the star $K_{(1,n)}$".

3.7. **Definition.** "The caterpillar is just a tree with the virtue of retaining a route even when all of its leaves are removed.".

3.8. **Definition.** "The graph is called a flower graph if it includes n vertices that form a n cycle and n sets of $m-2$ vertices that form $m-cycles$ around the $n-cycles$ so that each m cycle uniquely crosses with the $n-cycles$ on a single edge. The graph is indicated by the symbol $f_{(n\times m)}$, which has $n(m-1)$ vertices and nm edges. The petals and centre of $f_{(n\times m)}$ are termed the $m-cycle$ and $n-cycle$, respectively. The n vertices that make up the centre are all of degree 4, whereas the remaining vertices are all of degree 2."

4. Composite Labelling of Some Graphs

4.1. **Theorem.** The $K_{1,n-1}$ star graph allows for composite labelling.

proof: Acycle linked graph with n vertices and $n-1$ edges is known as a star graph H. Let the vertices be $v_1, v_2, ..., v_n$ and the edges be $u_1, u_2, ..., u_m$. Let v_n be the central vertex. It is worth noting that $|V(H)| = n$ and $|E(H)| = n-1$. $f : (V(H) \bigcup E(H)) \rightarrow 1, 2, 3, ..., 2n-1$ to create a bijection labelling for all i by $f(v_i) = 2i-1$ and $f(v_i v_n) = 2i$ for all i.

Clearly $gcd(f(uv), (vw)) \neq 1$. As a result, the above function gives composite labelling for a graph H. That is, the $K_{1,n-1}$ star graph is a composite labeling.

Example

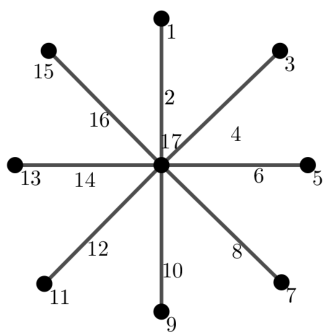

FIGURE 1. Composite labelling of Star graph $k_{1,n-1}$

4.2. **Theorem.** The crown graph $c_n o k_1$ admit composite labelling.

proof: Let $H = c_n o k_1$ be the crown graph, with $v_1, v_2, ..., v_n$ representing the vertices and $u_1, u_2, ..., u_m$ representing the edges. We can see that $|V(H)| = n$ and $|E(H)| = m$. As follows, create a bijection labelling $f : (V(H) \bigcup E(H)) \to 1, 2, 3, ..., m + n)$ by $f(v_i) = 2i - 1$ for all i and $f(v_i v_n) = 2i$ for all i. where $1 \leq i \leq n(n + 1)$

Clearly $gcd(f(uv), (vw)) \neq 1$. Thus the defined function provides composite labeling for a graph. That is, the crown graph $c_n o k_1$ is a composite labelling.

Example

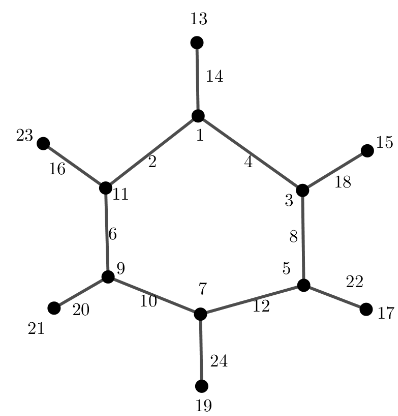

FIGURE 2. Composite Labelling of $C_8 o k_1$

Example

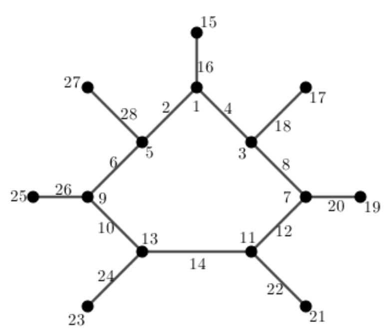

FIGURE 3. Composite Labelling of $C_7 o k_1$

4.3. **Theorem.** The comb graph $P_n o K_1$ admits composite labeling.
proof: $H = P_n o K_1$ is a comb graph with $2n$ vertices and $(2n-1)$ edges. Let's call the vertices $v_1, v_2, ..., v_n$ and the edges $u_1, u_2, ..., u_m$. Note that, $|V(H)| = 2n$ and $|E(H)| = 2n - 1$

To label a bijection $f:(V(H)\bigcup E(H))\rightarrow 1,2,3,...,$ as follows by $f(v_i)=2i-1$ for all i and $f(v_iv_n)=2i$ for all i. where $1\leq i\leq n(n+1)$

Clearly $gcd(f(uv),(vw))\neq 1$. Thus the function defined above provides composite labeling for a graph G. That is, comb graph P_noK_1 is a composite labelling.

Example

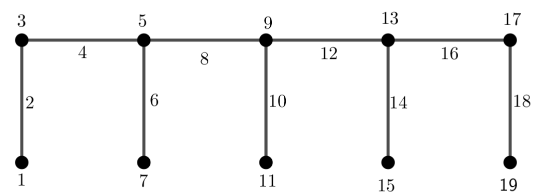

FIGURE 4. Composite labelling of P_5ok_1

4.4. **Theorem.** The Bistar graph $B_{(m,m)}$ admits composite labeling.

proof:The Bistar graph has $(2m+2)$ vertices and $(2m+1)$ edges. The bistar network $B_{(m,m)}$ has precisely two vertices of degree m. Let $v_1,v_2,....,v_n$ and $u_1,u_2,....,u_m$ be the edges. $|V(H)|=2m+2$ and $|E(H)|=2m+1$, as can be shown. Create a bijection labelling $f:(V(H)\bigcup E(H))\rightarrow 1,2,3,...,4m+3$ by $f(v_i)=2i-1$ for all i and $f(v_iv_n)=2i$ for all i. where $1\leq i\leq n(n+1)$.gcd(f(uv),(vw))$\neq$ 1 is undeniably true. As a result, the defined function gives composite graph labelling. $B_{(m,m)}$, for example, is a composite labelling.

Example

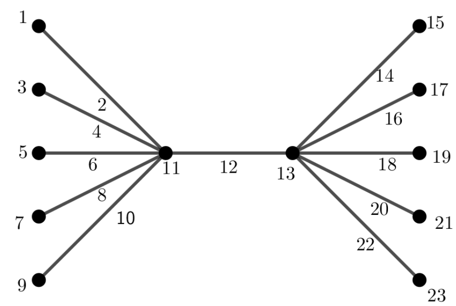

FIGURE 5. Composite labelling of Bistar graph($B_{5,5}$)

4.5. **Theorem.** The join sum of two copies of cycles c_n admits composite labeling. proof:let H be sum of two copies of cycle (c_n) have n-vertices and m-edges.let $v_1, v_2, ..., v_n$ denote the set of vertices and let $u_1, u_2, ..., u_m$ denote the edges. Note that $|V(G)| = n$ and $|E(G)| = n + 1$

A bijection labeling defined as $f : (V(G) \bigcup E(G)) \rightarrow 1, 2, 3, ..., 2n + 1$.

$$f(v_i) = \begin{cases} f(V_i) = 2i - 1 & \text{if} i = 1, 3, ...n \\ f(V_i)) = n + m & \text{if} i = 2 \end{cases}$$

Let e_1 be the central edges of e_2 and e_3 and e_1 be ending vertices having degree 2.

$$g(E_i) = \begin{cases} f(V_iV_i + 1) = 3i & \text{if} i = 1, 2 \\ f(V_iV_i + 1) = i(i + 1) & \text{if} i = 3 \end{cases}$$

The remaining edges are label as even integers.(i.e) f(V_iV_j) $= 2n$.

Case 1: If $n \equiv 0(mod3)$.

Example

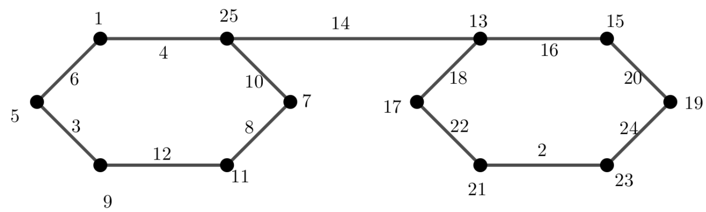

FIGURE 6. Cycle Graph of C_6

Case 2: If $n \equiv 0(mod4)$.

Example

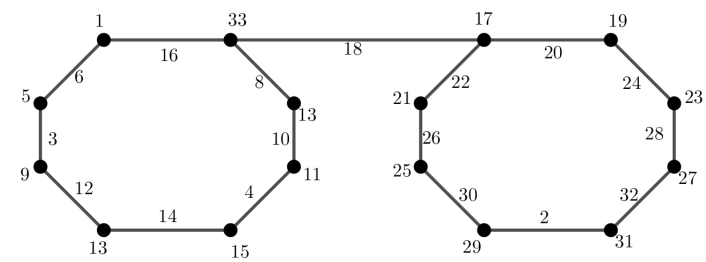

FIGURE 7. Cycle Graph of C_8

Case 3: If $n \equiv 1(mod4)$. **Example**

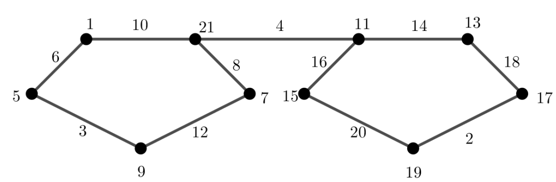

FIGURE 8. Cycle Graph of C_5

4.6. **Theorem.** One point union of six copies of p_4 admit composite labelling.

proof: Let H be a graph with a route p_n of length n and $k_{(1,t)}$.Let v_1 be the central vertex for p_n^t and $v_{l,m}(1 \leq l \leq t, 1 \leq m \leq n)$ be the consecutive vertices of each branch of p_n^t from v_1.

As follows, create a bijection labelling $f : (V(H) \bigcup E(H)) \rightarrow 1, 2, 3, ..., m+n$ by $f(v_i) = 2i - 1$ for all i and $f(v_i v_n) = 2i$ for all i. where $1 \leq i \leq n(n+1)$.

Clearly $gcd(f(uv),(vw)) \neq 1$. As a result, the above function gives composite labelling for a graph H. That is, the One point union of six copies of p_4 is a composite labeling.

Example

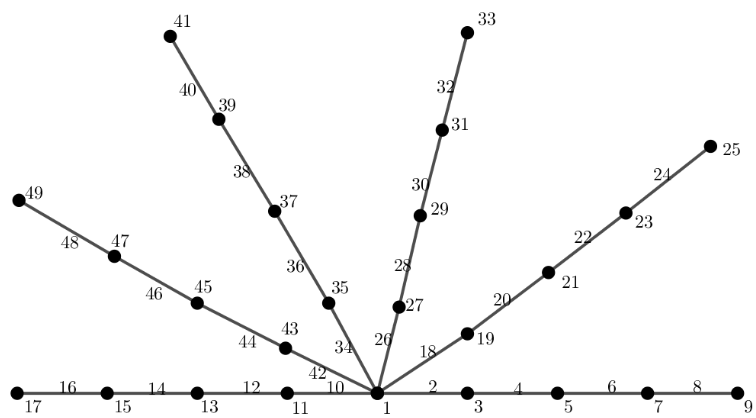

FIGURE 9. Composite Labelling of P_4

4.7. **Theorem.** All caterpillar trees are composite labelling.

proof:A caterpillar is a tree with the property that if all of the leaves are removed, a trail remains. Let the vertices be $v_1, v_2, ..., v_n$ and the edges be $u_1, u_2, ..., u_m$. We can see that $|V(H)| = m$ and $|E(H)| = n$

$f : (V(H) \bigcup E(H)) \rightarrow 1, 2, 3, ..., m + n$ is defined as follows by $f(v_i) = 2i - 1$ for all i and $f(v_i v_n) = 2i$ for all i. where $1 \leq i \leq n(n + 1)$.

Clearly $gcd(f(uv),(vw)) \neq 1$. Thus the defined function provides composite labeling for a graph. (i.e) caterpillar trees are composite labelling.

Example

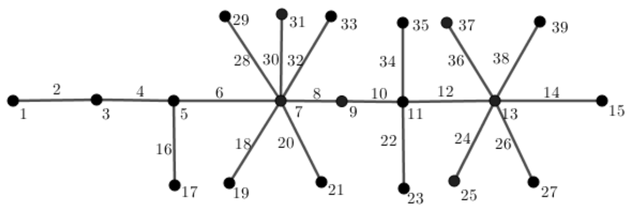

Figure 10. Composite Labelling of Caterpillar Tree

4.8. **Theorem.** The flower graph $f_{(n\times m)}$ admit composite labelling.

proof: Let G be a graph of a complete $f_{(nxm)}$ flower graph. Let $u_1, u_2, ..., u_m$ be the vertices and let $v_1, v_2, ..., v_n$ be the edges. We note that $|V(G)| = m$ and $|E(G)| = n$ A bijection labelling is defined as $f : (V(G) \bigcup E(G)) \to 1, 2, 3, ..., m + n$ as follows.

$$f(Vi) = \begin{cases} f(u_1) = 2i - 1 & \text{if} i = 1, 3, 4, 6, 7, 9, 11, 12, ...m \\ f(u_2) = 2n + 1 & \text{if} i = 2 \\ f(u_3) = 2n + 3 & \text{if} i = 5 \\ f(u_4) = 2n + 5 & \text{if} i = 8 \end{cases}$$

$$g(Ei) = \begin{cases} g(v_1) = 3i & \text{if} i = 1, 2, 3, 4, 5, 6 \\ g(v_2) = 2i & \text{if} i = 7, 8, 10, 11, 12, ...(n-3) \\ g(v_3) = i + 1 & \text{if} i = 9 \\ g(v_4) = i - 60 & \text{if } i = n - 2 \\ g(v_5) = (i-5)/5 & \text{if} i = n - 1 \\ g(v_6) = 2 & \text{if} i = n \end{cases}$$

Example

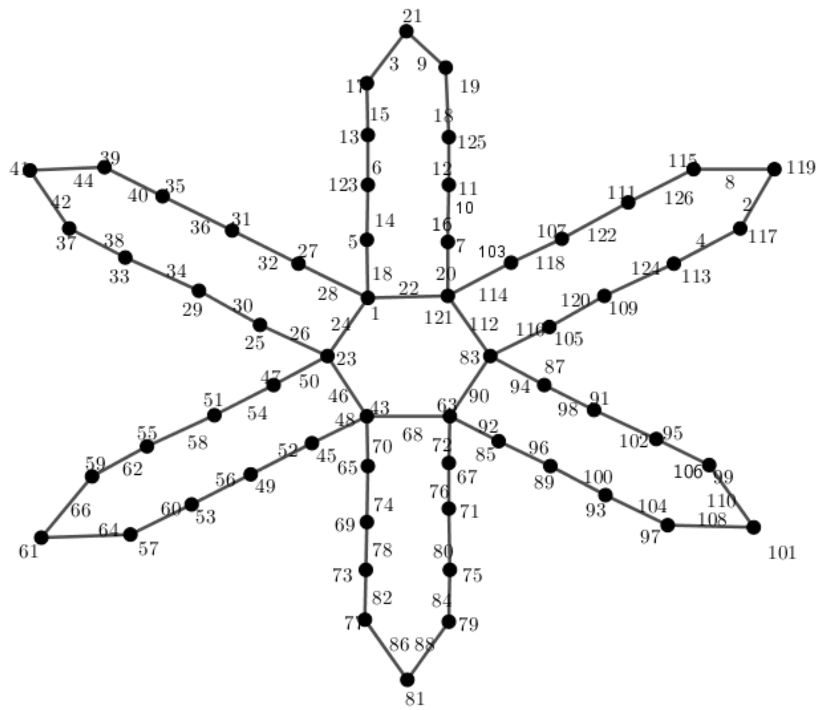

FIGURE 11. Composite Labelling of The flower graph

5. APPLICATION

5.1. **BLOCKCHAIN.** Distributed ledger technology (DLT) is a relatively recent technological breakthrough that has far-reaching implications for many sectors. Although cryptographic technologies have been the basis for blockchains for some time, their inclusion as a useful package is truly innovative. In the electrical field, the combination of distributed energy resources and the multiplication of grid-interacting devices excites blockchain potential. But blockchains are exactly what blockchains are basically unchangeable digital ledgers that can be used to securely record all transactions that take place on a given network, which cannot be changed once the data is sealed within a block. This includes not only financial transaction data but also anything of value. Technology enables a new world of decentralized communication and integration by creating the infrastructure to allow one to connect with one another securely, cheaply and quickly without a centralized intermediary.Its range is linked like a star map.

5.2. **NETWORK.** With a single wireless access point, more home networks and public WiFi hotspots may be connected (broadband router in the case of home networking). Instead, commercial computer networks set up several access points to cover a broader geographical region with their wireless network. Each access point has a limit on the number of connections it can accept as well as the quantity of network traffic it can manage. Multiple access points are connected into one huge network using a star graph with composite labelling Technic, demonstrating that the total size is enhanced.

5.3. **ONLINE SHOPPING.** If you have been considering and seeing an item on an online application and then decided not to purchase it, the message about that item will continue to be sent to you regardless of which App you use on your phone or laptop. They also bring and hold items that are linked in front of their sight. That item will circle back to you and remain there until you purchase it. As if it were a star graph, it continues circling back.

6. CONCLUSION

Its very Interesting to study graphs which admit of Composite labeling of various classes of graphs such as Star graph, The crown graph $(c_n \times k_1)$,The comb graph $(P_n \times K_1)$, The Bistar graph $B_{(n,n)}$, Join sum of two copies of cycle(c_n), Both copies of even cycles$c_n(n \geq 6)$ partaking a common line, single point union of six copies of p_4 , All caterpillar trees, Flower graph $f_{(n\times m)}$,The middle graph of path p_n are established. Composite labelling of other sorts of graphs is still a work in progress, and it will be completed later.

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