



Decision Support for Agri-Food Supply Chains in the E-Commerce Era: the in-Bound Inventory Routing Problem with Perishable Products

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Abstract. We consider an integrated planning problem that combines purchasing, inventory, and inbound transportation decisions in an agri-food supply chains where several suppliers (farmers) offer a subset of products with different selling prices and available quantities. We provide a mixed-integer programming formulation of the problem and a matheuristic decomposition that divides the problem into two stages. First, the purchasing and inventory problem is solved; second, the capacitated vehicle routing problem is solved using a split CVRP procedure. Computational experiments on a set of generated test instances show that the matheuristic can solve instances of large size within reasonably short computational times, providing better solutions than its MIP counterpart (in the absence of other approaches in the literature with which to make direct comparisons).

Keywords: Agri-food supply chain · inbound transportation · inventory routing problem · perishable products.

1 Introduction

The spread of online shopping through e-commerce platforms has disrupted not only traditional business models but also the supply chains that support them, with a growth of 7 to 10% in European countries in recent years [24]. Consumers now can access a global offer of products that can be delivered to any location within short times. In turn, producers (even small ones) can access competitive markets that were previously attainable only for large corporations with expensive infrastructure for distribution and marketing. These technology-based trade relationships have increased democratization in access to markets and provided efficiencies and convenience for both consumers and producers.

The agriculture sector has especially benefited from these technology-based business models, as small farmers can move away from intermediaries which traditionally provided distribution channels but take a large share of the revenue of the end-markets [11]. E-commerce platforms, although intermediaries as well,

provide more transparent relationships with final customers, such as restaurants or hotels demanding unique characteristics from specific producers (e.g., organic, fair-trade) at competitive prices. These advantages contribute to sustainability in global commerce by closing historical gaps in competitiveness between small and large players [20].

The shift towards a market based on several small producers implies coordinating a two-echelon supply chain with a network of participants (rather than a single provider). First, there is an echelon in which products are collected from suppliers and taken to a distribution center where inventories are managed (first mile). Then, there is an echelon in which products are distributed to end customers (last mile). The design and operation of efficient supply chains is crucial to enabling more competitive markets, in contrast to traditional structures characterized by a concentration of large agricultural companies [18].

The supply chains induced by e-commerce in the agricultural sector have special features whose treatment is incipient in the literature. The joint treatment of procurement logistics and inventory management (i.e., first mile logistics) has been little studied. Often, problems assume there is a supplier that guarantees the provision of products under a direct delivery, instead of addressing the logistics of picking up products from distributed suppliers with changing prices and availability. Moreover, the fact that such procurement strategy must respond to a dynamic demand of perishable products is a challenging realistic feature that has not been considered in the literature. In practice, companies struggle to coordinate procurement strategies with inventory management of fresh agricultural products. For example, perishable food waste in 2017 reached losses of 47 billion USD per year in China and 218 billion USD in the United States [13]. Therefore, solving such integrated problem efficiently is paramount to achieve the benefits associated to e-commerce.

The objective of this research is to develop and test algorithms that can efficiently support first-mile logistics decisions as part of a decision support system for agri-food supply chains in the context of e-commerce. At this stage, we present a two-stage math-heuristic methodology that integrates decisions about quantities to purchase of each of a set of products, inventory levels to satisfy the demand of perishable products, as well as selection of suppliers and routing of vehicles to replenish products at a warehouse. Section 2 provides an overview of the literature on this problem, while Sections 3 and 4 detail the characteristics of the problem and the solution approach, respectively. Section 5 provides a set of computational experiments and analysis, and Section 6 concludes.

2 Literature Review

The majority of the research that considered integrated inventory management and routing decisions are focuses on outbound routing problem, which is most commonly referred to as the Inventory Routing Problem, and the most studied variant in the literature is known as the Vendor Managed Inventory (VMI) problem, in which customers transfer the responsibility of inventory management to

a vendor, who knows the stock levels of their customers and must plan a distribution scheme to maintain adequate levels for all products of all customers. A general review of the transportation IRP is presented in [1] and [6].

Few authors address the First-mile problem with inbound transportation and inventory decisions, as most problems assume that ordered products simply arrive at the warehouse, disregarding the selection of suppliers and the logistics of collecting products from them. [16] and [17] consider a multiperiod, multi-supplier, many-to-one supply chain structure problem with a single assembly plant in which each supplier provides a distinct part type. In both cases, the problem is deterministic, and the solution approach is approximate optimization. In [7] a decomposition matheuristic is developed to solve an assembly, production, inventory routing problem with inbound transportation. The authors must select the supplier to visit, the order, and the inventory level at the supplier and the plant. The supplier offers only one type of product. After, in [8] the authors solve the same problem, but at this time, the suppliers have different products available. A B&C algorithm is proposed to solve the problem. In contrast to previous works, in our work the suppliers have a different capacity for each product in each period and to be able to pick up that product from the supplier you have to pay a price that varies according to the supplier and the period. In addition, the supplier's inventory cannot be managed. The work that most closely resembles ours is presented in [5], however, the authors proposed a non-linear model, test its performance on a single test instance, and they consider price discounts in the suppliers. There is some work that considers product perishability, inventory management and routing decisions together; however, the authors assume direct shipment from suppliers to the warehouse and do not consider the selling prices ([23] and [25])

In the first mile problem proposed in this research, the company must plan the procurement logistics (i.e., which suppliers to visit, in which order, and how much to buy of each product from each supplier) based on the estimated demands from customers, the current inventory levels, and the supplier characteristics (location, as well as product prices and availability). The Multi-Vehicle Traveling Purchaser Problem (MV-TTP) addresses this specific challenge (See [19]). MV-TTPs, can be classified according to the following four categories referring to the available supply, demand, vehicle capacity, and purchasing policy, as is mentioned in [4].

The Table 1 presents a comparison between the different MV-TTP variants with our work.

Table 1. MV-TTP variants comparison

	non-split	non-split	split	split	
unrestricted	[2], [9], [10], [12],[21],[22], [4]		invalid		capacited
unrestricted	[3]		invalid		uncapacited
restricted	invalid	our work	invalid	[9], [14]	capacited
restricted	invalid	[15]	invalid	[3]	uncapacited
	unitary	general	unitary	general	

The focus of this paper at hand is inbound transportation corresponding a restricted, capacitated, general Multi Vehicle PP with non-split purchases plus inventory management of perishable products at the warehouse.

3 Problem definition and mathematical formulation

The in-bound multi-product inventory routing problem (IB-MP-IRP) addressed in this work consists of a many-to-one system composed of a set of M suppliers and a single warehouse. Over the discrete periods t a planning horizon T , the warehouse satisfies a deterministic demand, $d_{k,t}$, of the k products in set K . The products are purchased and collected from the geographically dispersed suppliers using a homogeneous fleet F of vehicles v located at the warehouse, each with capacity Q . The suppliers must be visited by only one vehicle, and the total quantities purchased in any supplier must not exceed the vehicle capacity (i.e., non-split constraints are considered). At period t , product k can be purchased from a subset of suppliers $M_{k,t} \subseteq M$; each supplier i has their own selling price $p_{i,k,t}$ and available quantity $q_{i,k,t}$ of each product. At each period, the warehouse can purchase more than demanded of any product and store the remaining units in inventory to supply future demand. This encourages a holding cost $h_{k,t}$. The warehouse has unlimited storage capacity. Each product has a perishable nature represented by the subset O_k that limits the maximum number of periods that the product can remain in inventory. We define the problem on a complete undirected graph with nodes set $N = M \cup \{0\}$, where 0 represent the warehouse, and a set of edges $E = \{(i, j) : i, j \in N, i < j\}$.

The decisions to make are: the quantity to be purchased of each product at each supplier and each period; the quantity to maintain in inventory of each product at the end of each period; the selection of suppliers to be visited; and the order in which each vehicle visits suppliers in each period (i.e., the routes). The warehouse needs to simultaneously minimize the purchasing, inventory, and transportation costs for the entire planning horizon. It is easy to show that the (IB-MP-IRP) is NP-hard since the Multi-Vehicle Traveling Purchaser Problem (TPP) is a special case of it for each period. The problem can be formulated as the following mixed-integer program:

Variables

- $I_{k,t,o}$: inventory level of product k of age o at the end of period t ($o = 0$ indicates the product is fresh, whereas $o = |O_k|$ is the latest age acceptable for product k)
- $r_{k,t}$: quantity of product k to be replenished at period t
- $y_{k,t,o}$: quantity of product k of age o to be shipped at period t
- $x_{i,j,t,v} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by vehicle } v \text{ at period } t \\ 0 & \text{otherwise} \end{cases}$
- $w_{i,t,v} = \begin{cases} 1 & \text{if supplier } i \text{ is visited by vehicle } v \text{ at period } t \\ 0 & \text{otherwise} \end{cases}$
- z_{ikt} : quantity of product k purchased at the supplier i at period t

Objective function

$$\min \sum_{t \in T} \left(\sum_{v \in F} \left(\sum_{(i,j) \in E} c_{ij} x_{ijtv} + \sum_{k \in K} \sum_{i \in M_k} p_{ikt} z_{ikt v} \right) + \sum_{k \in K} \sum_{o \in O_k} h_{kt} I_{kto} \right) \quad (1)$$

Subject to

$$I_{k1o} = I_{k0o} - y_{k1o} \quad , \forall k \in K, \forall o \in O_k | o > 1 \quad (2)$$

$$I_{kt1} = r_{kt} - y_{kt1} \quad , \forall k \in K, \forall t \in T \quad (3)$$

$$I_{kto} = I_{kt-1o-1} - y_{kto} \quad , \forall k \in K, \forall t \in T | t > 1, \forall o \in O_k | o > 1 \quad (4)$$

$$\sum_{o \in O_k} y_{kto} = d_{kt} \quad , \forall k \in K, \forall t \in T \quad (5)$$

$$\sum_{v \in F} \sum_{i \in M_{kt}} z_{ikt v} = r_{kt} \quad , \forall k \in K, \forall t \in T \quad (6)$$

$$\sum_{v \in F} z_{ikt v} \leq q_{ikt} \quad , \forall k \in K, \forall t \in T, \forall i \in M_{kt} \quad (7)$$

$$z_{ikt v} \leq q_{i,k,t} w_{itv} \quad , \forall k \in K, \forall t \in T, \forall v \in F \quad (8)$$

$$\sum_{v \in F} w_{itv} \leq 1 \quad , \forall t \in T, \forall i \in M \quad (9)$$

$$\sum_{k \in K} \sum_{i \in M_{kt}} z_{ikt v} \leq Q \quad , \forall t \in T, \forall v \in F \quad (10)$$

$$\sum_{(i,j) \in \delta^+(\{m\})} x_{ijtv} = \sum_{(i,j) \in \delta^-(\{m\})} x_{ijtv} = w_{m,t,v} \quad , \forall v \in F, \forall t \in T, \forall m \in M \quad (11)$$

$$u_{itv} - u_{jtv} + |N| x_{ijtv} \leq |N| - 1 \quad , \forall t \in T, \forall v \in F, \forall i \in M, \forall j \in M \quad (12)$$

$$I_{kto} \geq 0 \quad , \forall k \in K, \forall t \in T, \forall o \in O_k \quad (13)$$

$$r_{kt} \geq 0 \quad , \forall k \in K, \forall t \in T \quad (14)$$

$$y_{kto} \geq 0 \quad , \forall k \in K, \forall t \in T, \forall o \in O_k \quad (15)$$

$$x_{i,j,t,v} \in \{0, 1\} \quad , \forall (i,j) \in E, \forall v \in F, \forall t \in T \quad (16)$$

$$w_{i,t,v} \in \{0, 1\} \quad , \forall i \in M, \forall v \in F, \forall t \in T \quad (17)$$

$$z_{ikt} \geq 0 \quad , \forall i \in M_{k,t}, \forall k \in K, \forall t \in T \quad (18)$$

The objective function (1) minimizes the total purchasing, inventory, and transportation costs. The holding cost is only considered in the warehouse. Initialization and inventory flow balance for the products of different ages is imposed through constraints (2)-(4). Constraint (5) guarantees demand satisfaction. Constraints (6) and (7) ensure to buy the quantity to be replenished and respect the quantities available from each supplier. Constraint (8) limits the quantity to be purchased at a supplier depending on the capacity of the vehicle that visits them. Constraints (9) and (10) limit the supplier to be visited only by one vehicle and not purchase more than vehicle capacity. These are the non-split constraints. Constraints (11) and (12) rule the visiting tour feasibility. Eqs. (11) impose that, for each visited supplier, exactly one arc must enter and leave the relative node, where, for any subset N' of nodes, $\delta^+(N') := \{(i, j) \in E : i \in V', j \notin V'\}$ and $\delta^-(N') := \{(i, j) \in E : i \notin V', j \in V'\}$. Inequalities (12) are connectivity constraints that prevent the creation of sub-tours by controlling the order of visits of the suppliers. Miller-Tucker-Zemlin (MTZ) formulation is used. The constraints (13)-(18) correspond to the domain of the variables.

4 A two-stage matheuristic decomposition

In this section, we present a two-stage matheuristic decomposition for the IB-MP-IRP with perishability. Algorithm 1 presents an overview of the matheuristic. Our algorithm decomposes the problem into two separate subproblems. The first subproblem aims to decide, for each period, the inventory levels and the quantity of each product to be purchased at each supplier by solving a simplified problem where an approximate transportation cost ($c_{i,t}^{\hat{}}$) is used to estimate the actual cost of visiting supplier i at period t . This is done as routing decisions are not considered at this stage. The objective function presented in (1) is reformulated as follows:

$$\min \sum_{t \in T} \left(\sum_{i \in M} \hat{c}_{it} w_{it} + \sum_{k \in K} \sum_{i \in M_k} p_{ikt} z_{ikt} + \sum_{k \in K} \sum_{o \in O_k} h_{kt} I_{kto} \right) \quad (19)$$

We define the first stage model with the objective function (19) subject to constraints (2)-(10), omitting the index $v \in F$ corresponding to the fleet of vehicles. Solving this model (line 3 - Algorithm 1) results in a (sub-optimal) purchasing and inventory plan that respects perishability.

The second stage solves, for each period t , a Capacitated Vehicle Routing Problem using the purchasing decisions found in the first stage. We fix the variables values of $\bar{w}_{i,t}$, $\bar{z}_{i,k,t}$. First, with the values of $\bar{w}_{i,t}$, a Nearest Neighbour Algorithm is run to obtain the order in which selected suppliers will be visited (line 4). Then, with this tour and the quantities to be purchased at each supplier, $\bar{z}_{i,k,t}$, a split C-VRP procedure is developed to obtain the vehicle routes that respect vehicle capacities (line 5). The augmented graph is built and the shortest path problem is solved using the Bellman-Ford algorithm. the solution

is assembled (line 5) with the routes and the values of z_{ikt} , w_{it} and I_{kto} , and if the solution is better than the incumbent, it is updated.

The information flow between the two stages is through parameter $\hat{c}_{i,t}$, which must be updated at each iteration. At iteration 0 ($iter = 0$), in line 1 (Algorithm 1) this parameter is initialized with the direct shipping cost (i.e., $\hat{c}_{i,t}^{iter} = c_{0,i} + c_{i,0}, \forall i \in M, \forall t \in T$). At the end of each iteration, the cost $\hat{c}_{i,t}^{iter}$ is updated after vehicles' routes have been obtained for each period (line 7). There are two ways of updating this parameter. First, if supplier i is part of a route at period t , the cost of the visiting them in the next iteration ($iter = iter + 1$) will be $\hat{c}_{i,t}^{iter} = (\hat{c}_{i,t}^{iter-1} + c_{i_p,i} + c_{i,i_s} - c_{i_p,i_s})/2$, where i_p and i_s are the predecessor and successor nodes of supplier i in their current route in that period. Second, if node i is not visited in any of the routes, then we set $\hat{c}_{i,t}^{iter} = (\hat{c}_{i,t}^{iter-1} + c_{insertion})/2$, where $c_{insertion}$ is equal to the cost of the cheapest insertion into an existing route in that period. This is based on the assumption that when a supplier i is eliminated from their route, an acceptable route can be obtained by connecting the predecessor and successor suppliers. Similarly, when inserting supplier i , an acceptable route can be obtained with the best insertion among all the routes in a specific period. The two stages are executed until the stopping criterion is reached (line 2).

Algorithm 1 Two-stage matheuristic decomposition

```

1: Initialize  $\leftarrow \hat{c}_{it}$ 
2: while termination condition not satisfied do
3:    $z_{ikt}, I_{kto}, w_{it} \leftarrow \text{SolvePurchaseAndInventory}(T, M, K, O_k, p_{ikt}, q_{ikt}, h_{kt}, \hat{c}_{it}; Q)$ 
4:    $Tour \leftarrow \text{NearestNeighbourAlgorithm}(T, w_{it}, c_{ij})$ 
5:    $Routes \leftarrow \text{SolveSplit_CVRP}(Tour, c_{ij}, z_{ikt}, Q)$ 
6:    $CurrentSolution \leftarrow \text{assembleSolution}(Routes, z_{ikt}, w_{it}, I_{kto})$ 
7:    $\hat{c}_{it} \leftarrow \text{updatedRoutingEstimation}(Routes, c_{ij})$ 
8:   Update Incumbent if CurrentSolution is better
9: end while
10: return Incumbent

```

5 Computational Experiments

The MIP and the decomposition matheuristic were implemented in Python 3.7, with Gurobi 9.1.1 as a solver for exact models. All computational experiments were performed on a 2.11 GHz processor with 16GB of RAM. The termination condition of Algorithm 1 (line 2) is parameter $maxCount$, which defines a maximum number of consecutive iterations without incumbent solution improvement, and was set at 20 iterations.

5.1 Data sets

There are no available data sets for the IB-MP-IRP considered in this work. We built a data set of 240 instances, taking into account the inventory characteristics of [16] and supplier characteristics of [19]. The number of suppliers, products, and periods were set as $M \in \{10, 25, 50, 100, 150\}$, $K \in \{10, 25, 50, 100\}$ and $T \in \{5, 10, 21\}$. The supplier locations were generated in a $[0, 1000] \times [0, 1000]$ square according to a uniform distribution and routing costs c_{ij} as truncated Euclidean distances. Each product k at period t is associated with $|M_{kt}|$ randomly selected suppliers, where $|M_{kt}|$ is uniformly generated number in $[1, |N| - 1]$. Parameter q_{ikt} of offered quantities is randomly taken in $[1, 15]$. Parameter λ is used to control the number of suppliers in a feasible solution through the product demand $d_{kt} := [\lambda \max_{i \in M_{kt}} q_{ikt} + (1 - \lambda) \sum_{i \in M_{kt}} q_{ikt}]$, $\forall k \in K, \forall t \in T$, with $0 < \lambda < 1$. The lower the value of λ , the higher the number of suppliers in a solution; λ was set as $\lambda \in \{0.5, 0.9\}$. The selling price p_{ikt} , and the holding cost h_{kt} were uniformly generated in $[1, 500]$. The latest age acceptable for product k was uniformly generated in $[1, |T|]$. To find a feasible vehicle capacity Q , we solve a model with objective function $\min z = Q$ subject to (2)-(10), omitting the index $v \in F$ corresponding to the fleet of vehicles. The result of this model is a feasible capacity, which is multiplied by 1.2 and rounded up to avoid a hard constraint. The number of vehicle $v \in F$ in the fleet is obtained by $|F| = \lceil \sum_{t \in T} \sum_{k \in K} d_{k,t} / Q \rceil$. Finally, two replicates were generated for each combination of M, K, T y λ .

5.2 Results

Two time-limits were defined to test the performance of the MIP. Initially, we attempted to solve instances within 1800 seconds, but required an increase to 3600 seconds to obtain reasonable solutions for a larger number of instances (i.e., the time limit of 3600 seconds is used when the model does not find an integer solution in 1800 seconds, or the MIP GAP is greater than 10%). Table 2 presents the results obtained by the MIP and the proposed matheuristic decomposition. Instances that do not appear in the table cannot be compared because the model did not find an integer solution in 3600 seconds or the computer memory is not sufficient. As the MIP model could not find an optimum solution in 3600 seconds for any instances; the results presented in the MIP columns are the best integer solution found. Also, the *Bestbound* and the gap calculated by Gurobi ($GAP_{B\&b}$) are reported. The column Δ_{MIP-H} is the percentage difference between the solutions obtained for the both approaches. It is calculated as $\Delta_{MIP-H} = ((BKS_{MIP} - BKS_H) / BKS_{MIP}) * 100$.

Table 2 shows that when the size of the instance increased, the $GAP_{B\&B}$ of the model also increase, as expected, due to the limited capacity of the exact model to solve big instances. On the other hand, when the size instance increased, the proposed mathheuristic finds better solutions than the MIP with a lower CPU time required. These results provide a preliminary confirmation about the potential of the proposed methodology. Currently, adjustments on the proposed

strategies and more comprehensive experiments are being developed in order to determine, with better statistical significance, which variations of instances and strategies lead to better results.

Table 2. Comparison MIP and Matheuristic

M	K	T	λ	ID	MIP				Math-heuristic		
					BKS	Time (s)	BestBound	GAP _{B&B}	BKS	Time (s)	Δ_{MIP-H}
10	10	5	0.5	1	179999	1801	170842.56	5.09	180835	0.89	0.46
10	10	5	0.5	2	173967	1802	165152.84	5.07	174396	0.83	0.25
10	10	5	0.9	1	152601	1802	146504.75	4.00	153044	0.91	0.29
10	10	5	0.9	2	168418	1801	161358.37	4.19	170265	0.95	1.10
10	10	10	0.5	1	351791	3606	310593.85	11.71	350653	2.17	-0.32
10	10	10	0.5	2	334332	3605	301934.26	9.69	336872	2.33	0.76
10	10	10	0.9	1	287696	3605	264497.63	8.06	286580	2.95	-0.39
10	10	10	0.9	2	255647	1804	246408.63	3.61	257094	1.39	0.57
10	25	5	0.5	1	336682	1802	324702.13	3.56	341168	1.13	1.33
10	25	5	0.5	2	338057	1802	312685.40	7.51	341580	1.22	1.04
10	25	5	0.9	1	290924	1802	273539.65	5.98	293026	1.20	0.72
10	25	5	0.9	2	281152	1802	266002.45	5.39	284913	0.95	1.34
10	25	10	0.5	1	651117	3606	598765.96	8.04	655208	3.11	0.63
10	25	10	0.5	2	699029	3607	654,946	6	707306	2.97	1.18
10	25	10	0.9	1	613872	3608	575706.69	6.22	615916	2.77	0.33
10	25	10	0.9	2	646987	1806	610636.93	5.62	650546	3.16	0.55
25	10	5	0.5	1	313159	3621	209788.22	33.01	318562	3.25	1.73
25	10	5	0.5	2	285475	3619	241288.27	15.48	286671	6.76	0.42
25	10	5	0.9	1	155564	3614	126783.32	18.5	159081	4.33	2.26
25	10	5	0.9	2	214676	3619	133925.36	37.62	214455	7	-0.10
25	10	10	0.5	1	590851	3675	375950.68	36.37	569111	12.54	-3.68
25	10	10	0.9	1	334735	3689	189621.55	43.35	329694	11.91	-1.51
25	10	10	0.9	2	322966	3656	220217.06	31.81	321824	10.12	-0.35
25	25	5	0.5	1	557466	3621	462325.5	17.07	553392	4.05	-0.73
25	25	5	0.5	2	556578	3621	427084.79	23.27	564814	3.95	1.48
25	25	5	0.9	1	373589	3622	241736.5	35.29	370630	5.36	-0.79
25	25	5	0.9	2	343748	3622	236289.92	31.26	338638	10.01	-1.49
25	25	10	0.5	2	1224065	3680	844555.11	31	1138494	13.58	-6.99
25	25	10	0.9	2	700790	3676	493845.7	29.53	683151	10.39	-2.52
50	10	5	0.5	1	444768	3744	234986.9	47.17	443279	15.01	-0.33
50	10	5	0.5	2	553434	3756	260349.26	52.96	549946	13.01	-0.63
50	10	5	0.9	1	337031	3872	115652.24	65.69	317415	15.44	-5.82
50	10	5	0.9	2	180982	3693	102025.31	43.63	185137	11.57	2.30

The table 3 presents the average computation times required by the matheuristics for each of the combinations of M , K and T . It can be seen that the largest increase in time occurs as the number of periods increases, followed by the increase in the number of suppliers. Although the matheuristic is able to obtain results for instances with 100 suppliers and 100 products when 21 periods are

Table 3. Heuristic computational times

M	Average CPU time (s)														
	10			25			50			100			150		
K\T	5	10	21	5	10	21	5	10	21	5	10	21	5	10	21
10	1	2	13	5	10	46	14	61	211	76	355	13038	259	-	-
25	1	3	10	6	12	47	21	61	239	88	580	-	304	-	-
50	2	5	17	9	25	59	20	75	247	109	401	-	-	-	-
100	4	9	34	9	24	79	36	80	304	118	497	-	-	-	-

considered with 100 or more suppliers, the computer memory is not enough to solve the model.

6 Conclusions

We presented a math-heuristic decomposition approach for the in-bound multi-product inventory routing problem, responding to challenges of the agri-food supply chain in the context of e-commerce. The key contribution is the integration of inventory decisions to satisfy a demand of perishable products with procurement decisions, including a selection of supplier for each product (with varying locations and product availability and price) and routing decisions. As a first step, the proposed math-heuristic approach obtains good quality solutions within reasonable computation times, given the limitations of the exact model and the scarcity of approaches for the proposed problem in the literature. Current results provide an initial confirmation on the potential of the proposed approach. However, ongoing work is devoted to the execution of a more comprehensive set of computational experiments that allow to reach more solid conclusions regarding the variants of the approach that are better suited for each type of instance. The next step of the research is the incorporation of stochasticity in the demand and product prices and availability at the supplier, as well as the possibility to update decisions dynamically, as usually allowed in e-commerce platforms.

References

1. Andersson, H., Hoff, A., Christiansen, M., Hasle, G., Løkketangen, A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. *Computers Operations Research*, 37(9), 1515-1536.
2. Baldacci, R., Dell'Amico, M., González, J. S. (2007). The capacitated m-ring-star problem. *Operations research*, 55(6), 1147-1162.
3. Bianchessi, N., Mansini, R., Speranza, M. G. (2014). The distance constrained multiple vehicle traveling purchaser problem. *European Journal of Operational Research*, 235(1), 73-87.
4. Bianchessi, N., Irnich, S., Tilk, C. (2021). A branch-price-and-cut algorithm for the capacitated multiple vehicle traveling purchaser problem with unitary demand. *Discrete Applied Mathematics*, 288, 152-170.

5. Çabuk, S., Erol, R. (2019). Modeling and Analysis of Multiple-Supplier Selection Problem with Price Discounts and Routing Decisions. *Applied Sciences*, 9(17), 3480.
6. Coelho, L. C., Cordeau, J. F., Laporte, G. (2014). Thirty years of inventory routing. *Transportation Science*, 48(1), 1-19.
7. Chitsaz, M., Cordeau, J. F., Jans, R. (2019). A unified decomposition matheuristic for assembly, production, and inventory routing. *INFORMS Journal on Computing*, 31(1), 134-152.
8. Chitsaz, M., Cordeau, J. F., Jans, R. (2020). A branch-and-cut algorithm for an assembly routing problem. *European Journal of Operational Research*, 282(3), 896-910.
9. Choi, M. J., Lee, S. H. (2010, July). The multiple traveling purchaser problem. In *The 40th International Conference on Computers Industrial Engineering* (pp. 1-5). IEEE.
10. Gendreau, M., Manerba, D., Mansini, R. (2016). The multi-vehicle traveling purchaser problem with pairwise incompatibility constraints and unitary demands: A branch-and-price approach. *European Journal of Operational Research*, 248(1), 59-71.
11. Gu, W., Archetti, C., Cattaruzza, D., Ogier, M., Semet, F., Speranza, M. G. (2022). A sequential approach for a multi-commodity two-echelon distribution problem. *Computers Industrial Engineering*, 163, 107793.
12. Hoshino, E. A., De Souza, C. C. (2012). A branch-and-cut-and-price approach for the capacitated m-ring-star problem. *Discrete Applied Mathematics*, 160(18), 2728-2741.
13. Ji, Y., Du, J., Han, X., Wu, X., Huang, R., Wang, S., Liu, Z. (2020). A mixed integer robust programming model for two-echelon inventory routing problem of perishable products. *Physica A: Statistical Mechanics and its Applications*, 548, 124481.
14. Manerba, D., Mansini, R. (2015). A branch-and-cut algorithm for the multi-vehicle traveling purchaser problem with pairwise incompatibility constraints. *Networks*, 65(2), 139-154.
15. Manerba, D., Mansini, R. (2016). The nurse routing problem with workload constraints and incompatible services. *IFAC-PapersOnLine*, 49(12), 1192-1197.
16. Moin, N. H., Salhi, S., Aziz, N. A. B. (2011). An efficient hybrid genetic algorithm for the multi-product multi-period inventory routing problem. *International Journal of Production Economics*, 133(1), 334-343.
17. Mjirda, A., Jarboui, B., Macedo, R., Hanafi, S., Mladenović, N. (2014). A two phase variable neighborhood search for the multi-product inventory routing problem. *Computers Operations Research*, 52, 291-299.
18. Majluf-Manzur, Á. M., González-Ramirez, R. G., Velasco-Paredes, R. A., Villalobos, J. R. (2020, December). An Operational Planning Model to Support First Mile Logistics for Small Fresh-Produce Growers. In *International Conference of Production Research—Americas* (pp. 205-219). Springer, Cham.
19. Manerba, D., Mansini, R., Riera-Ledesma, J. (2017). The traveling purchaser problem and its variants. *European Journal of Operational Research*, 259(1), 1-18.
20. Prajapati, D., Chan, F. T., Daultani, Y., Pratap, S. (2022). Sustainable vehicle routing of agro-food grains in the e-commerce industry. *International Journal of Production Research*, 1-26.
21. Riera-Ledesma, J., Salazar-González, J. J. (2012). Solving school bus routing using the multiple vehicle traveling purchaser problem: A branch-and-cut approach. *Computers Operations Research*, 39(2), 391-404.

22. Riera-Ledesma, J., Salazar-González, J. J. (2013). A column generation approach for a school bus routing problem with resource constraints. *Computers Operations Research*, 40(2), 566-583.
23. Rohmer, S. U. K., Claassen, G. D. H., Laporte, G. (2019). A two-echelon inventory routing problem for perishable products. *Computers Operations Research*, 107, 156-172.
24. Viu-Roig, M., Alvarez-Palau, E. J. (2020). The impact of E-Commerce-related last-mile logistics on cities: A systematic literature review. *Sustainability*, 12(16), 6492.
25. Wei, M., Guan, H., Liu, Y., Gao, B., Zhang, C. (2020). Production, Replenishment and Inventory Policies for Perishable Products in a Two-Echelon Distribution Network. *Sustainability*, 12(11), 4735.