Analytically Calculate the Renormalized Electron and Electron Neutrino Chain Propagators in SM

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***Abstract*—The frame of un-renormalized electron chain propagator, which composed of different physical processes, is analyzed in the standard model, and the un-renormalized electron chain propagator is given through dimensional regularization method. The analytical expression of the renormalized electron chain propagator is obtained by introducing the counter term, and then absorbing the counter term to the "bared" un–renormalized quantity in the "on-shell" renormalization scheme. The analytical expressions of renormalized electron neutrino chain propagator is also obtained by the same treatment. Finally, the relative corrections of the chain propagators to the tree propagators are given in large scale of propagator energy. The results show that the correction of the chain propagators are reasonable.**

***Index Terms*—Standard Model; Electron chain propagator; Renormalization; Relative correction.**

I. INTRODUCTION

Electroweak Standard Model (SM) is the most influential phenomenological theory in particle physics. It has made great success in describing the weak and electromagnetic interactions, and the new physics predicted by SM have been confirmed in many experiments, especially the Higgs particle was founded at LHC[1][2]. Since the standard model has been proposed by Weinberg and Salam, it has aroused a serious of related theoretical studies. Meanwhile, the works involving the "precision" test of SM the theoretical earned special academic concern all along. These theoretical studies have proposed the development of SM. Moreover; using perturbative quantum field theory to do calculation is the most important works when we do theoretical computing.

Theoretical predictions should have an accuracy compared to or even better than the experimental errors. So we are forced to take into account high order corrections if we want to do accurate calculations. The contribution of "renormalized finite quantity" (radiation correction) is very small, but this small contribution is very important to the deep study of related physical problems[3][4][5][6][7][8]. , It is obviously can reflect "radiation correction" of physical matter more accurately if one can acquire the exact result of this tiny correction effectively. Furthermore, the exact results are also helpful to the study and discussion of physical problems in depth. Many important processes, such as Bhabha scattering[9][10][11][12][13]and Compton scattering[14][15][16][17][18], involve electron and electron neutrino propagators in particle physics.

In this paper, we will introduce the model of electron and electron neutrino chain propagators to research the interactions that involving electron and electron neutrino in SM. We analyze the framework of electron chain propagator, which composed of different physical loops in detail first. In addition, we acquire the renormalized electron chain propagator $S_{\mathrm{F},R}^{(\mathrm{chain})}(p)$ by introducing the counter terms in the on-shell scheme and absorbing the counter terms into the un-renormalized "bare" parameters; then we figure out the renormalized electron chain propagator $S_{\mathrm{F},R}^{(\mathrm{chain})}(p)$ by complex function integral method. Meanwhile, we also obtain the analytical expressions of renormalized electron neutrino chain propagator $S_{\nu,R}^{(\mathrm{chain})}(p)$ by the same treatment. Finally, the relative corrections of the electron and electron neutrino chain propagators to the tree propagators are given in large scale of propagator energy.

The rest of the paper is organized as follows. Section II gives the construction of the electron chain propagator. In Section III, the analytical and numerical results of the renormalized electron chain propagator are carried out. Section IV gives the analytical and numerical results of the renormalized electron neutrino chain propagator. Finally, we conclude the work in Section V.

II. THE CONSTRUCTION OF THE ELECTRON CHAIN PROPAGATOR

After spontaneous symmetry breaking the Lagrangian for the electron field in SM is [19]

$$\begin{aligned} L =& \frac{ie}{4s_W c_W}\bar{\psi}_e\gamma^\mu 2(c_V - c_A\gamma_5)\psi_e Z_\mu - e\bar{\psi}_e\gamma^\mu\psi_e A_\mu \\ &- \frac{e}{2\sqrt{2}s_W}\left[\bar{\psi}_e\gamma^\mu(1-\gamma^5)\psi_{\nu_e}W_\mu^-\right. \\ &\left.+ \bar{\psi}_{\nu_e}\gamma^\mu(1-\gamma^5)\psi_e W_\mu^+\right] \\ &- \frac{em_e}{2s_W m_W}\left[\left(\bar{\psi}_e\psi_e H + i\bar{\psi}_e\gamma_5\psi_e\phi_0\right)\right. \\ &\left.+ \frac{1}{\sqrt{2}}\,\bar{\psi}_e(1-\gamma_5)\psi_{\nu_e}\phi_-\right] \end{aligned} \tag{1}$$

By the interaction Lagrangian, the electron self-energy involves 6 different physical processes in SM that shown in Fig.1.

Science the electron participates different physical interactions in SM, its chain propagator $S_{\mathrm{F}}^{(\mathrm{chain})}(p)$ contains complex internal electroweak interactions. The construction of $S_{\mathrm{F}}^{(\mathrm{chain})}(p)$ can be found in Fig.2.

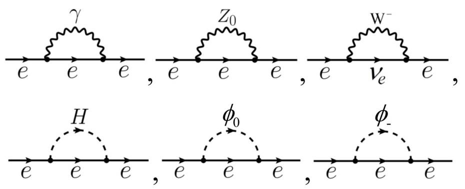

Fig. 1. Electron self-energy loops in SM.

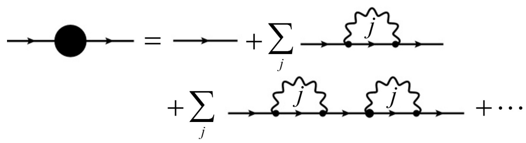

Fig. 2. The construction of Feynman diagram for electron chain propagator in SM.

By Fig.2, the electron chain propagator $S_{\mathrm{F}}^{(\mathrm{chain})}(p)$ can be expressed as

$$\begin{aligned} S_F^{(\mathrm{chain})}(p) &= S_F(p)\cdot\sum_{n=0}^{\infty}\left[-i\Sigma(p)\cdot S_F(p)\right]^n \\ &= S_F(p)\cdot\frac{1}{1+i\Sigma(p)\cdot S_F(p)} \end{aligned} \tag{2}$$

Where, $\Sigma(p)=\Sigma^{\gamma}(p)+\Sigma^{\mathrm{Z}}(p)+\Sigma^{\mathrm{W}}(p)+\Sigma^{\mathrm{H}}(p)+\Sigma^{\phi_0}(p)+\Sigma^{\phi_\pm}(p)$.

There will appear an infinite number of complex internal interactions when we consider chain propagator, one need to introduce the counter term to eliminate the divergence: $-i\hat{\Sigma}(p)=-i\Sigma^{(2)}(p)+(-i\delta\Sigma^{(2)}(p))$. Thus the renormalized electron chain propagator can be expressed as

$$S_{F,R}^{(\mathrm{chain})}(p)=S_F(p)\cdot\frac{1}{1+i\hat{\Sigma}(p)\cdot S_F(p)} \tag{3}$$

So the count of the renormalized finite quantities of electron chain propagator attributes to the count of the 6 loop divergences in Fig.1. Their sum general expression can be expressed as

$$\Sigma(p)=\hat{p}\Sigma_V(p)+\hat{p}\gamma_5\Sigma_A(p)+m_e\Sigma_S(p) \tag{4}$$

Specifically, $\Sigma_V(p)$, $\Sigma_A(p)$ and $\Sigma_S(p)$ are given by

$$\begin{aligned} \Sigma_V(p) = \frac{\alpha}{4\pi}\Big[&\big(2B_0(p^2,\lambda,m_e)+2B_1(p^2,\lambda,m_e)-1\big) \\ &+\frac{\left(1-4s_W^2\right)^2+1}{16s_W^2c_W^2}\big(2B_1(p^2,m_e,m_Z)+1\big) \\ &+\frac{1}{4s_W^2}\big(2B_1(p^2,0,m_W)+1\big) \\ &+\frac{m_e^2}{4s_W^2m_W^2}\big(B_1(p^2,m_e,m_H) \\ &+B_1(p^2,m_e,m_Z)+2B_1(p^2,m_\lambda,m_W)\big)\Big] \end{aligned} \tag{5}$$

$$\begin{aligned} \Sigma_A(p) = -\frac{\alpha}{4\pi}\Big[&\frac{1-4s_W^2}{8s_W^2c_W^2}\big(2B_1(p^2,m_e,m_Z)+1\big) \\ &-\frac{1}{4s_W^2}\big(2B_1(p^2,0,m_W)+1\big) \\ &+\frac{m_e^2}{2s_W^2m_W^2}B_1(p^2,m_\lambda,m_W)\Big] \end{aligned} \tag{6}$$

$$\begin{aligned} \Sigma_S(p) = \frac{-\alpha}{4\pi}\Big[&\frac{\left(1-4s_W^2\right)^2-1}{16s_W^2c_W^2}\big(4B_1(p^2,m_e,m_Z)-2\big) \\ &+\frac{m_e^2}{4s_W^2m_W^2}\big(B_1(p^2,m_e,m_Z)-B_1(p^2,m_e,m_H)\big) \\ &+\big(4B_0(p^2,\lambda,m_e)-2\big)\Big] \end{aligned} \tag{7}$$

Where, $s_W=\sin\theta_W$, $c_W=\cos\theta_W$, and θ_W is Weinberg angle, which defined as $\cos\theta_W = m_W/m_Z$. The photon contribution was calculated with a small photon mass λ in order to regularize possible infrared divergences. Besides, the two-point functions B_0 and B_1 are defined as

$$\begin{cases} B_0(p^2,m_1,m_2)=\Delta-\displaystyle\int_0^1 \ln\left[f(p,x)/M^2\right]dx \\ B_1(p^2,m_1,m_2)=-\dfrac{1}{2}\Delta+\displaystyle\int_0^1 x\ln\left[f(p,x)/M^2\right]dx \end{cases} \tag{8}$$

Where, $f(p,x) = p^2x(x-1)+m_1^2(1-x)+m_2^2x-i\varepsilon$, $\Delta = \frac{2}{\varepsilon}-\gamma_E+\ln 4\pi-\ln\left(M^2/\mu^2\right)$, μ is the renormalized scale, M is an arbitrary mass and its value does not influence the result of electron renormalized chain propagator in the on-shell scheme.

Instead of the vector and axial vector parts of the self-energies $\Sigma_{V,A}$ in Eq. (3), it may be more convenient to use the right- and left-handed parts

$$\Sigma(p)=\hat{p}\frac{1-\gamma_5}{2}\Sigma_L(p)+\hat{p}\frac{1+\gamma_5}{2}\Sigma_R(p)+m_e\Sigma_S(p) \tag{9}$$

Where, $\Sigma_R=\Sigma_V+\Sigma_A$ and $\Sigma_L=\Sigma_V-\Sigma_A$.

III. Analytical Results of The Renormalized Electron Chain Propagator

In this section, we will calculate the analytical result of renormalized electron chain propagator. One needs to calculate the two point functions $B_0(p^2,m_1,m_2)$ and $B_1(p^2,m_1,m_2)$ first. The analytic results read[20]

$$\begin{aligned} B_0(p^2,m_1,m_2) = \Delta-\Big\{&\ln\left(p^2/M^2\right)+\sum_{\lambda=1,2}\ln\left(1-x_\lambda\right) \\ &-x_\lambda\ln\left(1-1/x_\lambda\right)-1\Big\} \end{aligned} \tag{10}$$

$$B_1(p^2,m_1,m_2) = -\frac{\Delta}{2} + \frac{1}{2}\left\{\ln\frac{p^2}{M^2} + \sum_{\lambda=1,2}\ln(1-x_\lambda) - x_\lambda - {x_\lambda}^2 \ln\left(1-\frac{1}{x_\lambda}\right) - \frac{1}{2}\right\} \quad (11)$$

x_λ are the roots of the equation $p^2x^2+\left(m_2^2-m_1^2-p^2\right)x+m_1^2-i\varepsilon=0$:

$$\lim_{\varepsilon\to 0} x_\lambda(p^2,\varepsilon) = \frac{(p^2+m_2^2-m_1^2)}{2p^2} \pm \begin{cases} \sqrt{\frac{(p^2+m_2^2-m_1^2)^2-4p^2m_2^2}{4p^4}} \text{ for } p^2<0, \\ 0<p^2<(m_1-m_2)^2, p^2>(m_1+m_2)^2 \\ -i\sqrt{\frac{4p^2m_2^2-(p^2+m_2^2-m_1^2)^2}{4p^4}} \\ \text{for } (m_1-m_2)^2<p^2<(m_1+m_2)^2 \end{cases} \quad (12)$$

It is not difficult to calculate the derivative of B_0 and B_1

$$\frac{\partial B_0}{\partial p^2} = \frac{-1}{p^2(x_1-x_2)}\sum_{\lambda=1,2}\ln(1-x_\lambda) \cdot\left[-x_\lambda \ln(1-1/x_\lambda)-1\right] \quad (13)$$

$$\frac{\partial B_1}{\partial p^2} = \frac{1}{p^2(x_1-x_2)}\sum_{\lambda=1,2}\ln(1-x_\lambda) \cdot\left[-x_\lambda - {x_\lambda}^2\ln(1-1/x_\lambda)-1/2\right] \quad (14)$$

It is rather difficult to evaluate B_1 numerically if x_λ is too large. In practice we use the formula involving the logarithm if $|x_\lambda|<100$; for $|x_\lambda|\geq 100$ we express B_1 and $\frac{\partial B_1}{\partial p^2}$ as Eq. (15) and (16) after Taylor expansion of $\ln\left(1-\frac{1}{x_\lambda}\right) = -\sum_{n=1}^{\infty}\frac{1}{nx_\lambda^n}$.

$$B_1 = -\frac{1}{2}\Delta + \frac{1}{2}\left\{\ln\frac{p^2}{M^2} + \sum_{\lambda=1,2}\ln(1-x_\lambda) + \frac{1}{3x_\lambda} + \frac{1}{4x_\lambda^2} + \frac{1}{5x_\lambda^3} + \cdots\right\} \quad (15)$$

$$\frac{\partial B_1}{\partial p^2} = \frac{1}{p^2(x_1-x_2)}\sum_{\lambda=1,2}\ln(1-x_\lambda) \cdot\left(\frac{1}{3x_\lambda} + \frac{1}{4x_\lambda^2} + \frac{1}{5x_\lambda^3} + \cdots\right) \quad (16)$$

By introducing counter terms $\delta Z_L, \delta Z_R$ and δm_e in the on-shell scheme[21], the renormalized electron self-energies can be written as

$$\hat{\Sigma}(p) = \hat{p}\Sigma_V^R(p) + \hat{p}\gamma_5\Sigma_A^R(p) + m_e\Sigma_S^R(p) \quad (17)$$

Or

$$\hat{\Sigma}(p) = \hat{p}\frac{1-\gamma_5}{2}(\Sigma_L+\delta Z_L) + \hat{p}\frac{1+\gamma_5}{2}(\Sigma_R+\delta Z_R) + m_e\left(\Sigma_S + \frac{\delta m_e}{m_e} + \frac{1}{2}\delta Z_L + \frac{1}{2}\delta Z_R\right) \quad (18)$$

With

$$\delta Z_L = \mathrm{Re}\Sigma_L(m_e) + m_e^2\frac{\partial}{\partial p^2}\mathrm{Re}\left[\Sigma_L+\Sigma_R+2\Sigma_S\right]\big|_{p^2=m_e^2} \quad (19)$$

$$\delta Z_R = \mathrm{Re}\Sigma_R(m_e) + m_e^2\frac{\partial}{\partial p^2}\mathrm{Re}\left[\Sigma_L+\Sigma_R+2\Sigma_S\right]\big|_{p^2=m_e^2} \quad (20)$$

$$\delta m_e = -m_e\mathrm{Re}\left[\frac{1}{2}\Sigma_L(m_e) + \frac{1}{2}\Sigma_R(m_e) + \Sigma_S^f(m_e)\right] \quad (21)$$

It is not difficult to calculate the results of $\delta Z_L, \delta Z_R$ and δm_e through Eq.(5-7) and Eq.(10,11). The numerical results $\Sigma_V^R(p)$, $\Sigma_A^R(p)$ and $\Sigma_S^R(p)$ are shown in Fig.3 and Fig.4. Fig.3 shows the real parts corrections of electron self-energy, and Fig.4 shows the imaginary parts corrections of electron self-energy.

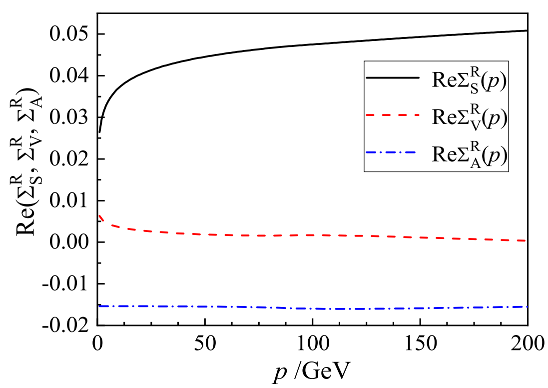

Fig. 3. Real parts corrections of electron self-energy.

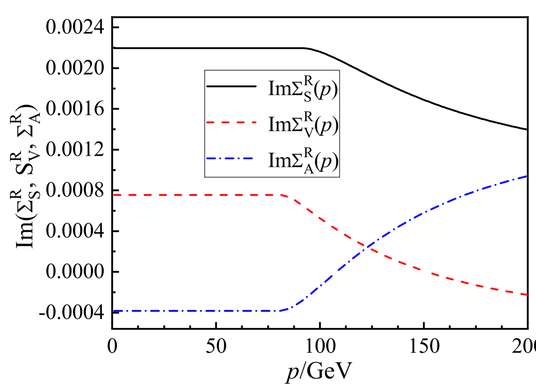

Fig. 4. Imaginary parts corrections of electron self-energy.

By introducing four parameters: $\Lambda(p)$, $\Omega(p)$, $\Upsilon(p)$ and $\Theta(p)$, we have

$$-i\hat{\Sigma}(p)\cdot S_F(p) = \Lambda(p) + \Omega(p)\hat{p} + \Upsilon(p)\gamma_5 + \Theta(p)\hat{p}\gamma_5 \quad (22)$$

With

$$\begin{cases} \Lambda(p) = \left[p^2\Sigma_V^R(p) + m_e^2\Sigma_S^R(p)\right]/\left(p^2 - m_e^2\right) \\ \Upsilon(p) = p^2\Sigma_A^R(p)/\left(p^2 - m_e^2\right) \\ \Theta(p) = m_e\Sigma_A^R(p)/\left(p^2 - m_e^2\right) \\ \Omega(p) = m_e\left[\Sigma_V^R(p) + \Sigma_S^R(p)\right]/\left(p^2 - m_e^2\right) \end{cases} \tag{23}$$

According to Eq.(22), the electron chain propagator also can be expressed as after introducing four corrective parameters

$$\begin{aligned} S_{F,\mathrm{R}}^{(\mathrm{chain})}(p) = & \frac{i\hat{p}}{p^2 - m_e^2}\xi_1(p) + \frac{im_e}{p^2 - m_e^2}\xi_2(p) \\ & + \frac{i\hat{p}\gamma_5}{p^2 - m_e^2}\xi_3(p) + \frac{i\gamma_5}{p^2 - m_e^2}\xi_4(p) \end{aligned} \tag{24}$$

With

$$\xi_1(p) = \frac{[(1-\Lambda) - m_e\Omega]F_1 + 2[(1-\Lambda)m_e - \Omega p^2]F_2}{{G_1}^2 - 4{F_2}^2p^2} \tag{25}$$

$$\xi_2(p) = \frac{[(1-\Lambda)m_e - \Omega p^2]F_1 + 2p^2[(1-\Lambda) - \Omega m_e]F_2}{({G_1}^2 - 4{F_2}^2p^2)m_e} \tag{26}$$

$$\xi_3(p) = \frac{(\Upsilon + m_e\Omega)F_1 + 2(\Theta p^2 - \Upsilon m_e)F_2}{{G_1}^2 - 4{F_2}^2p^2} \tag{27}$$

$$\xi_4(p) = \frac{(\Upsilon m_e + \Omega p^2)F_1 + 2p^2(\Theta m_e - \Upsilon)F_2}{{G_1}^2 - 4{F_2}^2p^2} \tag{28}$$

where $F_1 = (1-\Lambda)^2 + \Omega^2p^2 - \Upsilon^2 - \Theta^2p^2$, $F_2 = (1-\Lambda)\Omega - \Upsilon\Theta$ and $G_1 = (1-\Lambda)^2 - \Upsilon^2 + (\Omega^2 - \Theta^2)p^2$.

Compared to the electron tree propagator $S_F(p) = \frac{i\hat{p}}{p^2-m_e^2} + \frac{m_e}{p^2-m_e^2}$, the $\xi_{i=1,2,3,4}(p)$ in formula (23) can be interpreted as corrective parameters. Table I and Table II give the corrective parameters $\xi_1(p), \xi_2(p), \xi_3(p), \xi_4(p)$ for p^2.

It can be seen from table I that the relative correction of $\xi_1(p)$ and $\xi_2(p)$ is not more than 5%, which is accord with the magnitude of electroweak correction. It also can be seen from table II that $\xi_3(p)$ and $\xi_4(p)$ are very small, which means that although there are two extra terms ($\xi_3(p)$ and $\xi_4(p)$) in electron chain propagator compared to the tree propagator, their contributions are very small.

TABLE I
CORRECTIVE PARAMETERS $\xi_1(p), \xi_2(p)$ FOR p^2

$p^2(\mathrm{GeV}^2)$	$\xi_1(p)$	$\xi_2(p)$
-1000^2	$0.9890 - 0.0100i$	$0.9417 - 0.0033i$
-100^2	$0.9924 - 0.0101i$	$0.9531 - 0.0032i$
-10^2	$0.9952 - 0.0102i$	$0.9636 - 0.0030i$
-1	$0.9978 - 0.0102i$	$0.9741 - 0.0029i$
1	$0.9979 - 0.0070i$	$0.9740 - 0.0019i$
10^2	$0.9952 - 0.0069i$	$0.9635 - 0.0018i$
100^2	$0.9929 - 0.0071i$	$0.9534 - 0.0016i$
1000^2	$0.9890 - 0.0100i$	$0.9417 - 0.0005i$

TABLE II
CORRECTIVE PARAMETERS $\xi_3(p), \xi_4(p)$ FOR p^2

$p^2(\mathrm{GeV}^2)$	$10^2 \times \xi_3(p)$	$10^5 \times \xi_4(p)$
-1000^2	$-1.1541 + 0.4278i$	$1.8786 - 0.4996i$
-100^2	$-1.3518 + 0.4347i$	$1.3659 - 0.4911i$
-10^2	$-1.3821 + 0.4377i$	$0.9449 - 0.4822i$
-1	$-1.3897 + 0.4401i$	$0.5307 - 0.4729i$
1	$-1.3978 + 0.5756i$	$0.5415 - 0.2352i$
10^2	$-1.3908 + 0.5721i$	$0.9555 - 0.2399i$
100^2	$-1.4389 + 0.8304i$	$1.3403 - 0.2501i$
1000^2	$-1.1651 + 0.2606i$	$1.8776 - 0.3968i$

IV. THE RENORMALIZED ELECTRON NEUTRINO CHAIN PROPAGATOR

The electron neutrino self-energy involves 3 different physical processes in SM. Similar to the calculation of the renormalized electron self-energy, we can also obtain the renormalized electron neutrino self-energy

$$\hat{\Sigma}^\nu(p) = \hat{p}\frac{1-\gamma_5}{2}(\Sigma_L^\nu(p) + \delta Z_L^\nu) \tag{29}$$

Where,

$$\begin{aligned} \Sigma_L^\nu(p) = & -\frac{\alpha}{4\pi}\left\{\frac{1}{2s_W^2}\left(2 + \frac{m_e^2}{m_W^2}\right)\left(B_1(k^2, m_e, m_W) + 1\right)\right. \\ & \left. + \frac{1}{4s_W^2c_W^2}\left(2B_1(k^2, 0, m_Z) + 1\right)\right\} \end{aligned} \tag{30}$$

The renormalized electron neutrino self-energy can be written as formula (30) in the on-shell condition scheme

$$\hat{\Sigma}_L^\nu(p) = \Sigma_L^\nu(p) - \mathrm{Re}\Sigma_L^\nu(0) \tag{31}$$

The tree propagator of electron neutrino propagator in SM is $S_\nu(p) = i\hat{p}/p^2$. The renormalized electron neutrino chain propagator can be expressed as

$$S_{\nu,R}^{(\mathrm{chain})}(p) = \frac{i\hat{p}}{p^2}\varsigma_1(p) + \frac{i\hat{p}\gamma_5}{p^2}\varsigma_2(p) \tag{32}$$

Where, $\varsigma_1(p) = \frac{2-\hat{\Sigma}_L^\nu(p)}{2-2\hat{\Sigma}_L^\nu(p)}$, $\varsigma_2(p) = \frac{\hat{\Sigma}_L^\nu(p)}{2-2\hat{\Sigma}_L^\nu(p)}$. Fig.5 and Fig.6 give the corrective parameters $|\varsigma_1(p)|$ and $|\varsigma_2(p)|$ for p.

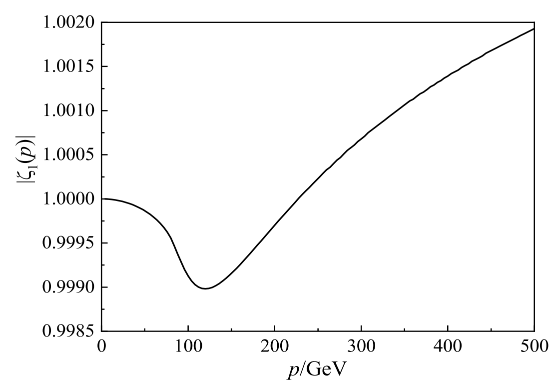

Fig. 5. The value of $|\varsigma_1(p)|$ with propagator energy.

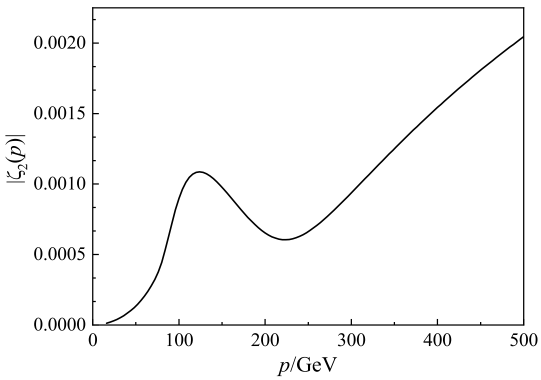

Fig. 6. The value of $|\varsigma_1(p)|$ with propagator energy.

V. Conclusions

In this paper, we analyzed and discussed the framework of electron and electron neutrino propagators and its renormalization in detail via SM. We obtained the analytical and numerical results at the same time, and the numerical results show that our results are in accordance with the magnitude of the electroweak correction. The renormalization model of this paper, not only taking into the "part infinite high order situation of perturbation theory, but the renormalized constants of counter terms into physical parameter (the mass of electron m) reasonably. Thus this renormalization and its count model undoubtedly has some theoretical significance academic reference.

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