A Novel Formulation for Multibody Systems with Unilateral Interactions Using Equimomental Systems of Point Masses

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Abstract

Mechanical systems models used for the time-domain simulation of contact problems lead to formulations that need to be solved. The accuracy of the solution is typically higher as the order of the integrator increases. Therefore, it would be ideal to use higher-order methods. However, it can be shown that for unilateral contact problems, first-order methods generally need to be used. Now, considering that only first-order precision is possible, the solution can be more or less accurate depending on how the physical system model is formulated.

One line of work looks at addressing this problem by separating the mechanical system model into smooth and nonsmooth parts and employing higher-order methods for the former [1]. However, the overall accuracy remains first-order for the nonsmooth part. Thus, the order of accuracy of the smooth and nonsmooth parts together is not improved. We address the problem here by proposing a new formulation for rigid body dynamics with unilateral interactions using equimomental systems of point masses (see Fig. 1). It is worth noting that some authors [3] have used point masses to model rigid body dynamics. However, to our knowledge, this approach has never been used for contact problems.

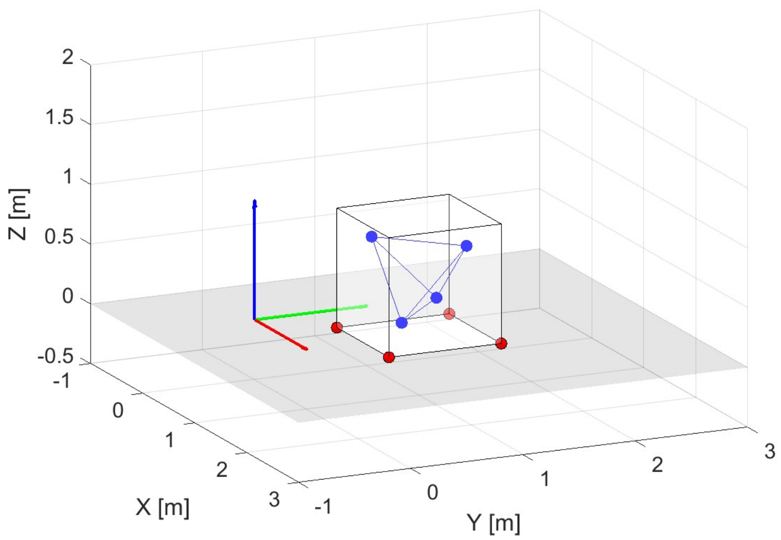

Figure 1: Rigid box modelled by an equimomental system of four point masses (in blue). In this example, contact is simulated through four contact points (in red).

The first element to consider when it comes to modelling rigid bodies with point masses is the number of those. A minimum of four point masses is needed to represent a rigid body through a system of point masses with the same inertial properties [2]. Therefore, we use four point masses in our model.

If we use point masses, a main advantage can be that the parameterization of rotations is not needed; the formulation only has to address point displacement. The mass matrix can be constant in such a formulation and may not depend on the configuration. Then, Coriolis and centrifugal terms can be eliminated.

These properties and the elimination of the need to deal with rotations can be beneficial. But, there is also a complicating factor: the point masses used in the model must always be at a constant distance from each other. This condition makes it necessary to include bilateral constraints in the formulation. Since we are using four point masses, six independent constraints are introduced. Considering that $\mathbf{q}_i = [x_i\ y_i\ z_i]^{\mathrm{T}}$ is the array with the absolute coordinates of the i-th point mass, the bilateral constraints may be written at the position level as

$$\phi_k: \quad \sqrt{(\mathbf{q}_i - \mathbf{q}_j)^{\mathrm{T}}(\mathbf{q}_i - \mathbf{q}_j)} - L_{ij} = 0, \tag{1}$$

where ϕ_k indicates the k-th constraint equation, with $k = 1,...,6$, and L_{ij} the distance between masses i and j.

Using the absolute coordinates of the point masses as the generalized coordinates to parameterize the problem, $\mathbf{q} = [\mathbf{q}_1\ \mathbf{q}_2\ \mathbf{q}_3\ \mathbf{q}_4]^{\mathrm{T}}$, and their time derivatives as the generalized velocities, $\mathbf{v} = \dot{\mathbf{q}}$, the previous constraint equations may be expressed at the velocity level as $\mathbf{A}_{\mathrm{b}}\mathbf{v} = \mathbf{0}$, with $\mathbf{A}_{\mathrm{b}}$ the bilateral constraint Jacobian.

Regarding the unilateral interactions, we can write three equations for the velocity of a given contact point using the velocities of three different point masses. By manipulating those equations, we can obtain the relation between the velocity of the contact point and the velocities of the point masses. Then, we can express the normal velocity components of the contact points with the generalized velocities of the model as $\mathbf{u}_{\mathrm{n}} = \mathbf{A}_{\mathrm{n}}\mathbf{v}$, with $\mathbf{u}_{\mathrm{n}}$ the normal contact velocities, and $\mathbf{A}_{\mathrm{n}}$ the normal contact Jacobian. A complementarity condition between the normal contact velocities, $\mathbf{u}_{\mathrm{n}}$, and the normal contact forces, $\boldsymbol{\lambda}_{\mathrm{n}}$, must be included to describe adequately the contact. The unilateral constraint conditions can then be written as $\mathbf{0} \leq \mathbf{u}_{\mathrm{n}} \perp \boldsymbol{\lambda}_{\mathrm{n}} \geq \mathbf{0}$, where the operator $\perp$ denotes component-wise complementarity.

Based on the above, we can establish the dynamics formulation as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} = \mathbf{f}_{\mathrm{a}} + \mathbf{A}_{\mathrm{b}}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{b}} + \mathbf{A}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{n}} \\ \mathbf{A}_{\mathrm{b}}\mathbf{v} = \mathbf{0} \\ \mathbf{0} \leq \mathbf{u}_{\mathrm{n}} \perp \boldsymbol{\lambda}_{\mathrm{n}} \geq \mathbf{0} \end{cases}, \tag{2}$$

where $\mathbf{M}$ is the generalized mass matrix, $\dot{\mathbf{v}}$ represents the generalized accelerations, $\mathbf{f}_{\mathrm{a}}$ the generalized applied forces, and $\boldsymbol{\lambda}_{\mathrm{b}}$ the generalized bilateral constraint forces.

We can use a time-stepping algorithm to discretize and solve this contact problem. This leads to an impulse-momentum level formulation that can be written as

$$\begin{cases} \begin{bmatrix} \mathbf{M} & -\mathbf{A}_{\mathrm{b}}^{\mathrm{T}} & -\mathbf{A}_{\mathrm{n}}^{\mathrm{T}} \\ \mathbf{A}_{\mathrm{b}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{\mathrm{n}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}^+ \\ h\boldsymbol{\lambda}_{\mathrm{b}}^+ \\ h\boldsymbol{\lambda}_{\mathrm{n}}^+ \end{bmatrix} + \begin{bmatrix} -\mathbf{M}\mathbf{v} - h\mathbf{f}_{\mathrm{a}} \\ h\dot{\mathbf{A}}_{\mathrm{b}}\mathbf{v} \\ h\dot{\mathbf{A}}_{\mathrm{n}}\mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_{\mathrm{n}}^+ \end{bmatrix} \\ \mathbf{0} \leq \mathbf{u}_{\mathrm{n}}^+ \perp \boldsymbol{\lambda}_{\mathrm{n}}^+ \geq \mathbf{0} \end{cases}, \tag{3}$$

where the time steps of size h are taken from time t^- to time t^+, and $\dot{\mathbf{A}}_{\mathrm{b}}$ and $\dot{\mathbf{A}}_{\mathrm{n}}$ are the time derivatives of the bilateral and normal constraint Jacobians, respectively. Here, terms with superscript $^+$ are evaluated at time t^+, while non-superscripted terms at time t^-.

This work presents a novel approach to model multibody systems with contact. The presented formulation can improve the accuracy of the smooth and nonsmooth parts together by representing rigid bodies with systems of point masses. It is worth noting that this is achieved by using only first-order integration methods.

References

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