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Static of mechanical behavior of Sandwich FGM beams structure under mechanical loads

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Abstract

This paper presents a static analysis of Sandwich Functionally Graded Material (FGM) beams structure under static mechanical loads. The structure of the beam is a sandwich structure. The material properties of Sandwich FGM beams are developed based on the principle of continuously changing material properties according to a defined rule. Analysis of mechanical behavior of Sandwich FGM beams structure based on Euler-Bernoulli beams theory and Timoshenko beams theory. Apply two beams theories to the illustrative problem, thereby giving the appropriate theory for each case.

1 Introduction

An object that has one dimension greater than the other (three dimensions) and is able to withstand bending is called a beam. The normal sandwich structure consists of two shells with higher stiffness than the core layer. Functionally graded material is a kind special composites with constantly changing material properties and are usually composed of two material components such as metal and ceramic are very common. It was developed by Japanese scientists in 1984. FGM withstands harsh environments such as high temperature, abrasion, high acid corrosion, and significantly reduces stress concentration problems.[1] The reason is to solve the large difference between the thermal expansion coefficients of the component materials, thereby forming cracks, causing destruction. Therefore, it has many potential applications for high-tech industries. Sandwich FGM beams structure is a beam. It has a sandwich construction with two thinner shells made of primary load-bearing FGM and a core made of a soft, lightweight homogenous material withstands shear, sound and heat insulation. This paper presents a sandwich beam FGM whose mechanical properties vary from only one direction (1D-FGSW).

2 Material and Methods

Two beam theories satisfy the basic assumptions and concepts of environmental mechanics continuously like three-dimensional Euclidean space, time are absolute, macro phenomenon, continuity of the environment. It is studied and analyzed according to the three basic equations of continuum mechanics, expressed through kinematics, constitutive equation and equilibrium. For the mathematical model of the Euler-Bernoulli beams theory (shear rigid beams), the cross-sectional plane is always perpendicular   to the beam axis. Timoshenko beams theory (shear flexible beams), the cross-section rotates at an angle   with respect to the plane perpendicular to the axis of the beam when there is a shear strain. [2]:
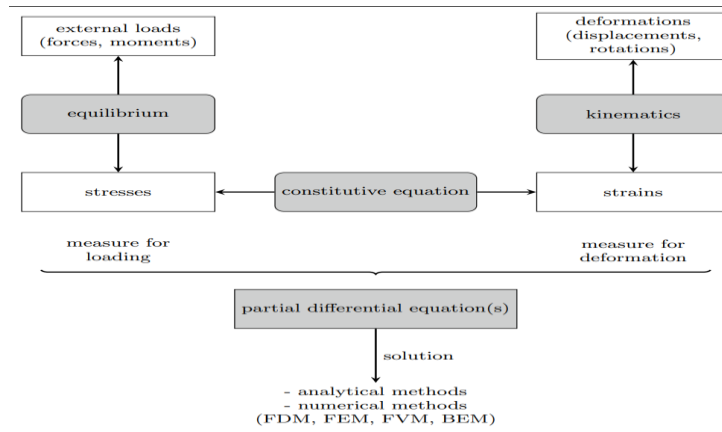
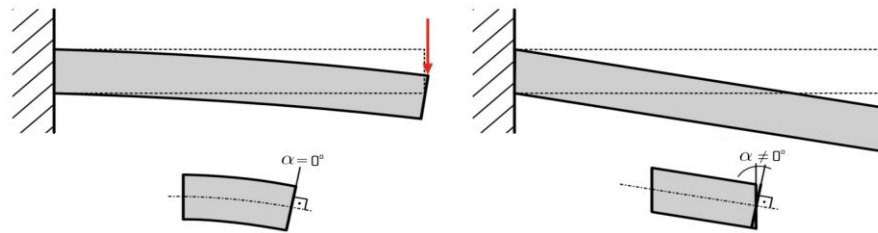**Figure 1.** Continuum mechanical modeling
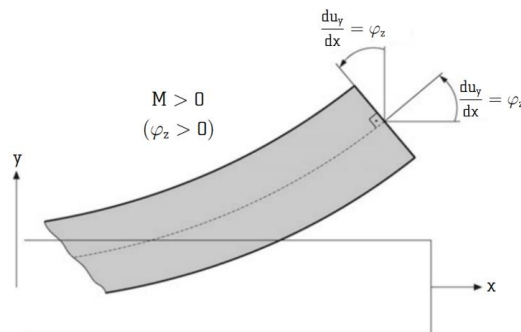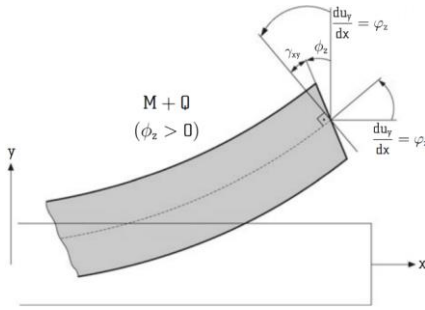
Figure 2. Continuum mechanical modeling

Figure 3. The kinematics relation for bending in the x-y plane (Euler Bernoulli beams theory)

Figure 4. The kinematics relation for bending in the x-y plane (Euler Bernoulli beams theory)

2.1 Euler-Bernoulli Beams Theory

Euler-Bernoulli beams theory is a special case of Timoshenko beams theory first proposed in 1750. It was used to build the Eiffel Tower, the Ferris wheel and fueling the second industrial revolution. Euler-Bernoulli beam or a thin is defined as a long prismatic body. The following derivations are restricted to some simplifications: Apply to straight beam only, the deformation is small and considered in a single plane, simple cross sections and bending is symmetrical, no elongation along and torsion around the longitudinal x axis (consider in the x-y plane). As a result, the basic equations according to the Euler-Bernoulli beam theory[2].

Differential equation:

$$EI_z \frac{d^4 y(x)}{dx^4} - q(x) = 0 \qquad (1)$$

Kinematics:

$$\varepsilon_x(x,y) = -y \frac{d^2[u_y(x)]}{dx^2} \qquad (2)$$

$$k(x) = -\frac{d^2[u_y(x)]}{dx^2} \qquad (3)$$

Constitutive equation:

$$\sigma_x(x,y) = E\varepsilon_x(x,y) \qquad (4)$$

2.2 Timoshenko Beams Theory

Analysis such as the Euler-bernoulli beams theory and consideration of the cross-section rotates at an angle with respect to the plane perpendicular to the axis of the beam[3]. The basic equations according to the Timoshenko beam theory will be obtained as follows:

Differential equation:

$$EI_z \frac{d^3 \varphi(x)}{dx^3} - q(x) = 0 \qquad (5)$$

3

$$\frac{dy(x)}{dx} + \left(\frac{EI_z}{k_s AG}\right)\frac{d^2\varphi(x)}{dx^2} - \varphi(x) = 0 \tag{6}$$

Kinematics:

$$\varepsilon_x(x,y) = -y\frac{d\phi_z(x)}{dx} \tag{7}$$

$$\phi_z = \frac{du_y(x)}{dx} - \gamma_{xy} \tag{8}$$

Constitutive equation:

$$\sigma_x(x,y) = E\varepsilon_x(x,y) \tag{9}$$

$$\tau_{xy}(x) = G\gamma_{xy}(x) \tag{10}$$

$$G = \frac{E}{2(1+??)} \tag{11}$$

3 The Illustrative Problem

A beam with description as picture and assuming L = 1 units, q= -1 units, ksAG = 1 unit and EIz = 1 unit. The convention that the left end of the beam are the coordinate axis. Find the deflection, rotation, bending moment, and shearing force as a function of the beam.
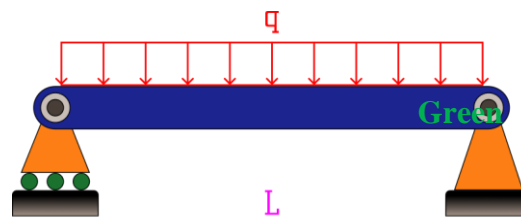

Figure 5. Simple Beam Math Problem

According to the Euler-Bernoulli beams theory.

Solve the differential equation of equilibrium (1) with four boundary conditions M=0 and y=0 on both the left and right ends of the beam and (2),(3),(4).

$$y = \left(\frac{1}{EI_z}\right)\left(\iiiint q(x)\,dx\,dx\,dx\,dx + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4\right) \tag{12}$$

$$\dot{y} = \left(\frac{1}{EI_y}\right)\left(\iiint q(x)dxdxdx + C_1\frac{x^2}{2} + C_2x + C_3\right)$$

(13)

$$\ddot{y} = \left(\frac{1}{EI_y}\right)\left(\iint q(x)dxdx + C_1x + C_2\right)$$

(14)

$$\dddot{y} = \left(\frac{1}{EI_z}\right)\left(\int q(x)dx + C_1\right)$$

(15)

$$\varphi = \dot{y}$$

(16)

$$M = EI_z\dot{y}$$

(17)

$$V = EI_z\ddot{y}$$

(18)

According to the Timoshenko beams theory.

Solve the differential equation of equilibrium (5) &(6) with four boundary conditions M=0 and y=0 on both the left and right ends of the beam and (7),(8),(9).

$$\varphi = \left(\frac{1}{EI_y}\right)\left(\iiint q(x)dxdxdx + C_1\frac{x^2}{2} + C_2x + C_3\right)$$

(19)

$$\dot{\varphi} = \left(\frac{1}{EI_z}\right)\left(\iint q(x)dxdx + C_1x + C_2\right)$$

(20)

$$\ddot{\varphi} = \left(\frac{1}{EI_z}\right)\left(\int q(x)dxdx + C_1\right)$$

(21)

$$\Delta = \left(\frac{-EI_z}{k_sAG}\right)\left(\iint q(x)dxdx + C_1x\right)$$

(22)

$$y = \left(\frac{1}{EI_z}\right)\left(\iiiint q(x)dxdxdxdx + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4 + \Delta\right)$$

(23)

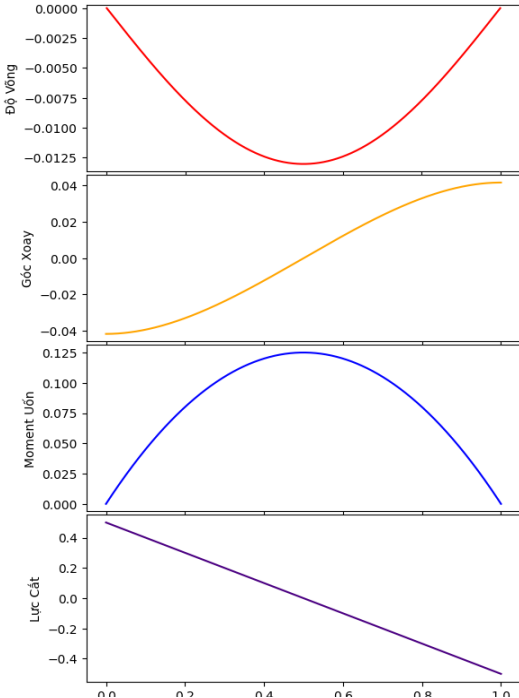$$M = EI_z\dot{\varphi}$$

(24)

$$V = EI_z\ddot{\varphi}$$
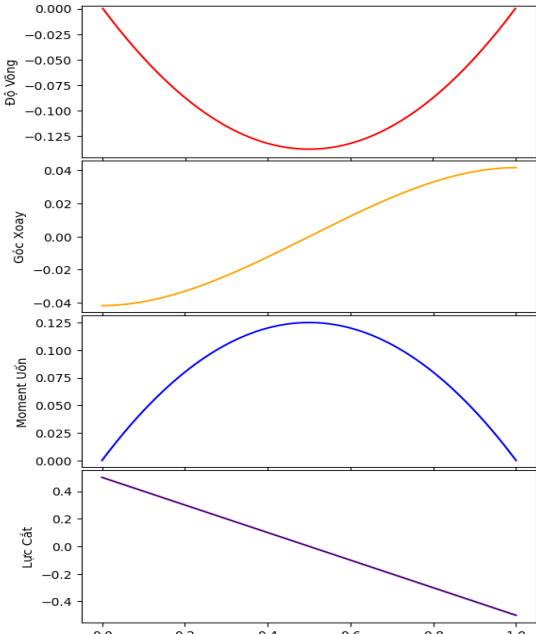
(25)

Figure 6. Euler-Bernoulli beams theory

Figure 7. Timoshenko beams theory

4 Conclusions

This study builds a first cheap prototype model to evaluate food quality, which can support the food industry in detecting irregular product by luminate the sample with multi-wavelength. This model has met several criteria such as simple setup, affordable price and portable. The result shows the trend of changing in the food properties under four illuminate condition in VIS-NIR band, instead of using invasive technique which damaged the product.

5 Acknowledgments

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